CHM225 Problem Set #1– September 10, 2008

1. A gas undergoes a reversible isothermal compression from a state 1 $= (T_1, V_1)$ to a state 2 $= (T_2, V_2)$. The equation of state of the gas is

$$\frac{PV^2}{(V+c)} = RT,$$

where c is a constant.

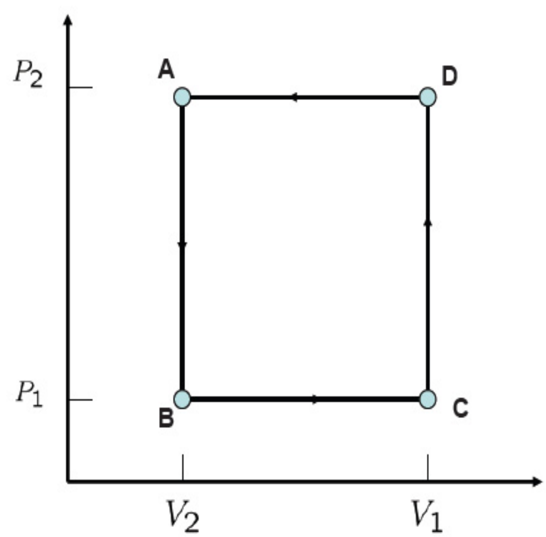
(i) Compute the work along this path from state 1 to state 2 .

(ii) Compute ΔU and q for this path from state 1 to state 2 .

(iii) Suppose the compression is carried out irreversibly at the external pressure P_2 corresponding to that of state 2 . What is work and how does it compare with that obtained in (i)?

(iv) Suppose the gas is ideal. What is the work for the isothermal compression taking the system from state 1 to state 2 and how does it compare to that obtained in (i)?

2. Suppose a system can exist in states $A = (P_2, V_2)$, $B = (P_1, V_2)$, $C = (P_1, V_1)$ and $D = (P_2, V_1)$. Consider the cyclic path $A \to B \to C \to D \to A$ shown in the figure.



(i) Compute the work in each step of the cycle.

(ii) For the entire cycle, compute the change in the internal energy, ΔU_{cycle}, the heat, q_{cycle}, and the work, w_{cycle}.

(iii) Suppose the cycle is traversed in the reverse direction, $A \to D \to C \to B \to A$? Compute the work in each step of the reversed cycle as well as ΔU_{cycle}, the heat, q_{cycle}, and the work, w_{cycle}.

3. One mole of CO_2 at 100^oC and 80 atm is compressed isothermally and reversibly to 100 atm. The pressure is then reduced to 80 atm by cooling reversibly at constant volume. Finally, the system returns to original state by warming reversibly at constant pressure. Assuming that CO_2 obeys the Van der Waals equation of state with constants $a = 3.592\ \ell^2\text{atm/mol}^2$ and $b = 0.04265\ \ell/\text{mol}$, calculate q and w for each step and for the cycle.

   You can assume $C_V = \frac{3}{2}R$. To find C_P use the relation

$$C_P - C_V = \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right]\left(\frac{\partial V}{\partial T}\right)_P.$$

   The partial derivatives on the right hand side must be calculated. To find $\left(\frac{\partial U}{\partial V}\right)_T$ use the relation

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P.$$

   You will have to use the Van der Waals equation to obtain unknown quantities that specify the different states. For example, the gas is initially at ($T_1 = 100$ C, $P_1 = 80$ atm). Use the Van der Waals equation to find V_1. This involves solving a cubic equation.

4. One mole of an ideal monotomic gas undergoes the following processes: 1. a reversible adiabatic expansion from (P_1, V_1, T_1) to (P_2, V_2, T_2). 2. Return to the initial state by a reversible straight line path where $P = cV + b$, where the constants b and c can be determined from the initial and final points of the path. Calculate ΔU, q and w for each step and for the cycle if $P_1 = 2$ atm, $T_1 = 0^oC$ and $V_2 = 2V_1$.

5. A gas has internal energy $U = \frac{5}{2}PV + c$, where c is a constant. Consider three states of the system where $A = (P_A, V_A)$, state B has the same pressure as state A and state C has the same volume as state A. The gas undergoes a reversible, cyclic process $A \to B \to C \to A$. The process $A \to B$ occurs at constant pressure P_A. The process $B \to C$ occurs along a path where the pressure varies linearly with volume. The process $C \to A$ occurs at constant volume.

   (i) Calculate q and w for each of the three steps in the cycle.

   (ii) Calculate q and w for a reversible process from A to B along a parabolic path where $P = P_A + c_2[V^2 - V(V_A + V_B) + V_AV_B]$ and c_2 is a constant.

   (iii) Find the forms of the curves $P = P(V)$ in the PV-plane along which $dq = 0$.