

## CHEM 225/221: PROBLEM SET 1

1. In this question,  $\hat{A}$  is an operator,  $f(x)$  is a function, and  $\alpha$  and  $\beta$  are constants. For each of the operators, determine whether  $\hat{A}$  is linear.

(a)  $\hat{A}f = \alpha f + \beta$

(b)  $\hat{A}f = \alpha d^2 f/dx^2$

(c)  $\hat{A}f = \sqrt{f}$

(d)  $\hat{A}f = d/dx f^2$

(e)  $\hat{A}f = xf$

2. Calculate the commutator of operators  $\hat{A}$  and  $\hat{B}$  defined by their operation on a function  $f$  (with constants  $\alpha$  and  $\beta$ ) as:

(a)  $\hat{A}f = df/dx$       $\hat{B}f = f^2$

(b)  $\hat{A}f = df/dx$       $\hat{B}f = d^2 f/dx^2$

(c)  $\hat{A}f = \alpha f + \beta$       $\hat{B}f = f^2$

(d)  $\hat{A}f = df/dx$       $\hat{B}f = f$

3. For the following linear operators  $\hat{A}$  acting on functions  $f$ , find the eigenvalues and eigenfunctions of  $\hat{A}$ :

(a)  $\hat{A}f = df/dx$

(b)  $\hat{A}f = d^4 f/dx^4$

(c)  $\hat{A}f = \alpha d^2 f/dx^2$