TERM TEST 1

Notes:

- The test has 2 total questions with each question containing multiple parts.
- Note the constants and integrals provided on page 2.
- Please show **all** work in test books.

1. (60 points) Consider a system of N non-interacting point particles in *two-dimensions*, each of which has a velocity with components (v_x, v_y) distributed according to

$$\phi_m(v_x) = \sqrt{\frac{\beta m}{2\pi}} e^{-\beta m v_x^2/2}.$$

   (a) (15 points) What is the probability density $G(v)$ for the speed of a particle $v = |\mathbf{v}|$, in the *two-dimensional* system. Note that $G(v)dv$ is the probability that any given particle in the system has a speed v in the range $[v, v + dv]$. *Warning:* This is not the same as in a $3 - D$ system.

   (b) (15 points) What is the most probable speed in the two-dimensional system?

   (c) (15 points) What is the probability density of energy per particle $P(\epsilon)$ for the $2 - D$ system?

   (d) (10 points) What is the average *total* energy of this system?

   (e) (5 points) What is the most probable energy per particle? How do you explain this result in light of your answer to $1 - d$.?

2. (40 points) Now consider a system consisting of N non-interacting point particles placed in a *cubic* container with sides of length L, with the origin at one of the corners of the box. Suppose the velocity components v_x, v_y and v_z of the particles are initially distributed according to the Maxwell distribution $\phi_m(v_x)$ and that particles interact with the walls via instantaneous hard collisions.

   (a) (10 points) Particles with what velocity component v_x are most likely to collide with the x-face at $x = L$ per unit time?

   (b) (10 points) Does the distribution of velocities change as a function of time? Why, or why not?

   (c) (15 points) Suppose a hole of area A_0 is poked at time $t = 0$ in each side of the faces at $x = 0$ and $x = L$. Derive and solve an equation for how the initial number of particles in the box with a velocity component v_x evolves as a function of time.

   (d) (5 points) Does the distribution of velocities in the container change in time after the holes have been poked? Justify your answer.

Integrals and Constants

$$N_A = 6.022142 \times 10^{23} \text{ molecules/mole} \qquad R = 8.3145 \text{ J/(mole-K)}$$

$$k = 1.38065^{-23} \text{ J/K}$$

$$\int_0^\infty x e^{-ax^2} dx = \frac{1}{2a}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int_{-\infty}^\infty e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty x^2 e^{-ax^2} dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty x^4 e^{-ax^2} dx = \frac{3}{8a^2}\sqrt{\frac{\pi}{a}}$$