Magnetic field symmetries of nonlinear transport with elastic and inelastic scattering

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We study nonlinear electronic transport symmetries in Aharonov-Bohm interferometers subjected to inelastic-scattering effects and show that odd (even) conductance terms are even (odd) in the magnetic field when the junction is (left-right) spatially symmetric. This observation does not hold when spatial inversion symmetry is broken, as we show numerically. Under elastic dephasing effects, the Onsager-Casimir symmetry is maintained beyond linear response, irrespective of spatial asymmetries.

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I. INTRODUCTION

The Onsager-Casimir symmetry relations[1] hold close to equilibrium, implying that the two-probe linear conductance is an even function of the magnetic field B. As a consequence, the two-terminal transmission function of coherent conductors satisfies $\mathcal{T}(B) = \mathcal{T}(-B)$. Within Aharonov-Bohm (AB) interferometers, this symmetry is displayed by the "phase rigidity" of the conductance oscillations with B.[2,3] Beyond linear response, the phase symmetry of the conductance, or reciprocity theorem, is generally not enforced, and several experimental works[4–8] have demonstrated its breakdown. Supporting theoretical studies have incorporated many-body interactions[9–13] but typically approached the problem by calculating the screening potential within the conductor self-consistently, a procedure often limited to low-order conduction terms.[9,11,12]

In this paper we aim to generalize the Onsager-Casimir relation of electrical conduction to the nonlinear transport regime, while allowing for inelastic-scattering effects. Phase-breaking and energy dissipation processes arise due to the interaction of electrons with other degrees of freedom, electrons, phonons, and defects. Here we incorporate such processes phenomenologically, by using the well-established and experimentally feasible method of Büttiker dephasing and voltage probes.[14,15] For spatially symmetric junctions we then discuss the exact symmetry relations beyond the Onsager symmetry and their violation, addressing all transport coefficients at the same footing. Expanding the current $I(\phi)$, passing across the interferometer, in powers of the bias $\Delta\mu$, we write

$$I(\phi) = G_1(\phi)\Delta\mu + G_2(\phi)(\Delta\mu)^2 + G_3(\phi)(\Delta\mu)^3 + \dots, \tag{1}$$

with $G_{n>1}$ as the nonlinear conductance coefficients. Here we have introduced the AB phase $\phi = 2\pi\Phi/\Phi_0$, Φ is the magnetic flux threading through the AB ring, and $\Phi_0 = h/e$ is the magnetic-flux quantum. In this work we study relations between two quantities: a measure for the magnetic-field asymmetry,

$$\Delta I(\phi) \equiv [I(\phi) - I(-\phi)]/2, \tag{2}$$

and the dc-rectification current,

$$\mathcal{R}(\phi) \equiv \tfrac{1}{2}[I(\phi) + \bar{I}(\phi)] = G_2(\phi)(\Delta\mu)^2 + G_4(\phi)(\Delta\mu)^4 + \dots, \tag{3}$$

with $\bar{I}$ defined as the current obtained upon interchanging the chemical potentials of the two terminals (assuming identical temperatures). We also study the behavior of odd conductance terms, $\mathcal{D}(\phi) \equiv G_1(\phi)\Delta\mu + G_3(\phi)(\Delta\mu)^3 + \dots$. For a noninteracting system we expect the relation $I(\phi) = -\bar{I}(-\phi)$ to hold. Combined with Eq. (1) we immediately note that $G_{2n+1}(\phi) = G_{2n+1}(-\phi)$ and $G_{2n}(\phi) = -G_{2n}(-\phi)$ with n as an integer. We show below that these relations are obeyed in a symmetric junction even when many-body interactions (inelastic scattering) are included, depending on both the applied bias, in a nonlinear manner, and the magnetic field, with no particular symmetry.

The principal results of this paper are now listed:

(i) Under elastic dephasing effects we prove that $\Delta I = 0$ and $\mathcal{R} = 0$, and thus the current displays an even symmetry with respect to the magnetic field.

(ii) We further incorporate inelastic effects and prove that in geometrically symmetric junctions $\Delta I(\phi) = \mathcal{R}(\phi) = -\mathcal{R}(-\phi)$.

(iii) Using a double-dot interferometer, we demonstrate with numerical simulations that under a spatial asymmetry both even and odd conductance terms show no particular magnetic-field symmetry; however, odd conductance terms show a weaker breakdown of symmetry. These observations corroborate with transport experiments on AB structures with apparently a geometrical asymmetry in the ring-lead coupling.[6]

Note that "spatial" or "geometrical" symmetry refers here to the left-right symmetry of the junction. For simplicity, we set $e = 1$, $h = 1$, and $k_B = 1$, and we ignore the electron spin.

II. BASIC EXPRESSIONS

Elastic dephasing effects are implemented here using the dephasing probe technique. Inelastic effects can be furthermore included adopting a voltage probe.[14] Particularly, we consider a setup including three terminals, L, R, and P, with the P terminal serving as the probe (see Fig. 1). Our analysis relies on two exact relations: Given the conservation of the

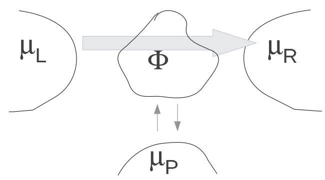

FIG. 1. Scheme of our setup. The horizontal arrow represents the charge current I. The two parallel arrows represent (same magnitude) currents into and from the P terminal, serving to induce elastic- and or inelastic-scattering effects.

total current and time reversal, the transmission coefficient from the ξ to the ν reservoir obeys the reciprocity relation:[14]

$$\mathcal{T}_{\xi,\nu}(\epsilon,\phi) = \mathcal{T}_{\nu,\xi}(\epsilon,-\phi), \quad \xi,\nu = L,R,P. \tag{4}$$

Furthermore, the total probability is conserved:[12]

$$\sum_{\xi\neq\nu} \mathcal{T}_{\xi,\nu}(\epsilon,\phi) = \sum_{\xi\neq\nu} \mathcal{T}_{\nu,\xi}(\epsilon,\phi). \tag{5}$$

We focus on the steady-state tunneling current from the L reservoir into the system, identifying $I(\phi) = I_L(\phi)$, where

$$I_L(\phi) = \int_{-\infty}^{\infty} d\epsilon[\mathcal{T}_{L,R}(\epsilon,\phi) f_L(\epsilon) - \mathcal{T}_{R,L}(\epsilon,\phi) f_R(\epsilon) + \mathcal{T}_{L,P}(\epsilon,\phi) f_L(\epsilon) - \mathcal{T}_{P,L}(\epsilon,\phi) f_P(\epsilon,\phi)], \tag{6}$$

written here assuming Landauer's picture of noninteracting electrons. The Fermi-Dirac distribution function $f_\nu(\epsilon) = [e^{\beta_\nu(\epsilon-\mu_\nu)}+1]^{-1}$ is defined in terms of the chemical potential μ_ν and the inverse temperature β_ν. In what follows we assume that the temperature is identical in all terminals. The current from the probe to the system is given by

$$I_P(\phi) = \int_{-\infty}^{\infty} d\epsilon[\mathcal{T}_{P,L}(\epsilon,\phi) f_P(\epsilon,\phi) - \mathcal{T}_{L,P}(\epsilon,\phi) f_L(\epsilon)] + \int_{-\infty}^{\infty} d\epsilon[\mathcal{T}_{P,R}(\epsilon,\phi) f_P(\epsilon,\phi) - \mathcal{T}_{R,P}(\epsilon,\phi) f_R(\epsilon)]. \tag{7}$$

The probe distribution function, generally phase dependent, is determined by the probe condition, as we explain below. For convenience, we simplify next our notation by dropping the reference to the energy of the incoming electron ϵ from both transmission functions and distribution functions and not putting limits of integrations, which are all evaluated between $\pm\infty$. When possible, we do not explicitly include ϕ in $\mathcal{T}_{\xi,\nu}$ and f_P, all evaluated at $+\phi$.[16] Using Eq. (6), we identify the deviation from the magnetic-field symmetry as

$$\Delta I = \int \frac{1}{2}[\mathcal{T}_{L,R} - \mathcal{T}_{R,L}] f_R d\epsilon + \int \frac{1}{2}[\mathcal{T}_{L,P} f_P(-\phi) - \mathcal{T}_{P,L} f_P(\phi)] d\epsilon. \tag{8}$$

The rectification contribution is written as

$$\mathcal{R} = \int \frac{\mathcal{T}_{P,L} - \mathcal{T}_{P,R}}{4}(f_L + f_R - f_P - \bar{f}_P) d\epsilon, \tag{9}$$

with $\bar{f}_P$ as the probe distribution when the biases μ_L and μ_R are interchanged.

III. FORMAL RESULTS

A. Elastic dephasing effects

We implement elastic dephasing demanding the energy-resolved particle current in the probe to diminish, $I_P(\epsilon) = 0$, with $I_P = \int I_P(\epsilon) d\epsilon$. Equation (7) provides the corresponding distribution:

$$f_P(\phi) = \frac{\mathcal{T}_{L,P} f_L + \mathcal{T}_{R,P} f_R}{\mathcal{T}_{P,L} + \mathcal{T}_{P,R}}. \tag{10}$$

As highlighted, this function depends on the magnetic flux. It is not difficult to prove that Onsager symmetry is satisfied here, beyond linear response. We plug f_P into Eq. (8) and find, after simple algebraic manipulations,

$$\Delta I = \frac{1}{2}\int [\mathcal{T}_{L,R} - \mathcal{T}_{R,L}] f_R d\epsilon + \frac{1}{2}\int \frac{[\mathcal{T}_{L,P}\mathcal{T}_{P,R} - \mathcal{T}_{P,L}\mathcal{T}_{R,P}] f_R}{\mathcal{T}_{P,R} + \mathcal{T}_{P,L}} d\epsilon. \tag{11}$$

Utilizing Eq. (5) in the form $\mathcal{T}_{L,P} = \mathcal{T}_{P,L} + \mathcal{T}_{P,R} - \mathcal{T}_{R,P}$, we organize the numerator of the second integral, $(\mathcal{T}_{P,R} - \mathcal{T}_{R,P})(\mathcal{T}_{P,R} + \mathcal{T}_{P,L}) f_R$. This results in

$$\Delta I = \frac{1}{2}\int [\mathcal{T}_{L,R} - \mathcal{T}_{R,L} + \mathcal{T}_{P,R} - \mathcal{T}_{R,P}] f_R d\epsilon, \tag{12}$$

which is identically zero, given Eq. (5). This concludes our proof that dephasing effects (implemented via a dephasing probe) cannot break the Onsager symmetry even in the nonlinear regime. Following similar steps we can show that $\mathcal{R} = 0$ under dephasing effects: We substitute f_P into Eq. (6) and obtain $I_L = \int [F_L f_L - F_R f_R] d\epsilon$ with $F_L = [\mathcal{T}_{L,R}(\mathcal{T}_{P,L} + \mathcal{T}_{P,R}) + \mathcal{T}_{L,P}\mathcal{T}_{P,R}]/(\mathcal{T}_{P,L} + \mathcal{T}_{P,R})$. F_R is defined analogously, interchanging L by R. Using Eq. (5), one can show that $F_L = F_R$, and thus $I = -\bar{I}$ and $\mathcal{R} = 0$. To conclude, $I = \mathcal{D}(\phi) = \mathcal{D}(-\phi)$ under elastic dephasing irrespective of spatial asymmetry.

B. Inelastic effects

We introduce both elastic and inelastic effects using the voltage probe technique, where we demand that the net-total particle current flowing in the P reservoir is zero, $I_P = 0$. This choice allows for energy exchange processes within the P reservoir. The probe condition produces the following relations:

$$\begin{aligned}
\int d\epsilon(\mathcal{T}_{P,L} + \mathcal{T}_{P,R}) f_P(\phi) &= \int d\epsilon(\mathcal{T}_{L,P} f_L + \mathcal{T}_{R,P} f_R),\\
\int d\epsilon(\mathcal{T}_{L,P} + \mathcal{T}_{R,P}) f_P(-\phi) &= \int d\epsilon(\mathcal{T}_{P,L} f_L + \mathcal{T}_{P,R} f_R),\\
\int d\epsilon(\mathcal{T}_{P,L} + \mathcal{T}_{P,R}) \bar{f}_P(\phi) &= \int d\epsilon(\mathcal{T}_{L,P} f_R + \mathcal{T}_{R,P} f_L).
\end{aligned} \tag{13}$$

We assume that the probe distribution follows a Fermi-Dirac form. One can obtain the respective unique[17] chemical potentials by solving these equations (separately) numerically using the Newton-Raphson method.[18]

First, we comment on the special point $\phi = 2\pi k$, k is an integer. When the magnetic phase is given by multiples of 2π, we note that $\mathcal{T}_{\nu,\xi} = \mathcal{T}_{\xi,\nu}$. Particularly, when $\mathcal{T}_{P,L} = \chi \mathcal{T}_{P,R}$, with χ an energy independent parameter, reflecting spatial asymmetry [see for example Eq. (18)], the probe condition Eq. (13) provides $\int \mathcal{T}_{P,\mu}(f_P + \bar{f}_P)d\epsilon = \int \mathcal{T}_{P,\mu}(f_L + f_R)d\epsilon$, $\mu = L,R$, resulting in $\mathcal{R} = 0$ in Eq. (9).

We now discuss symmetries when the junction has a left-right inversion symmetry. In this case the mirror symmetry $\mathcal{T}_{P,L}(\phi) = \mathcal{T}_{P,R}(-\phi)$ applies. This translates to the relation $\mathcal{T}_{P,L}(\phi) = \mathcal{T}_{R,P}(\phi)$. We plug this result into Eq. (13) and note that $\bar{f}_P(\phi) = f_P(-\phi)$. The deviation from phase rigidity, Eq. (8), can also be expressed as

$$\Delta I = \frac{1}{2}\int d\epsilon[(\mathcal{T}_{L,R} - \mathcal{T}_{R,L})f_L - \mathcal{T}_{R,P} f_P(-\phi) + \mathcal{T}_{P,R} f_P(\phi)]. \tag{14}$$

We now define ΔI by the average of Eqs. (8) and (14):

$$\Delta I = \frac{1}{4}\int d\epsilon[(\mathcal{T}_{L,R} - \mathcal{T}_{R,L})(f_L + f_R) + (\mathcal{T}_{L,P} - \mathcal{T}_{R,P})f_P(-\phi) + (\mathcal{T}_{P,R} - \mathcal{T}_{P,L})f_P(\phi)].$$

Using Eq. (5), we note that $\mathcal{T}_{L,R} - \mathcal{T}_{R,L} = \mathcal{T}_{P,L} - \mathcal{T}_{L,P}$. Furthermore, given that $\mathcal{T}_{P,L} = \mathcal{T}_{R,P}$ in geometrically symmetric junctions we get

$$\Delta I = \frac{1}{4}\int (\mathcal{T}_{P,L} - \mathcal{T}_{P,R})(f_L + f_R - f_P - \bar{f}_P)d\epsilon = \mathcal{R}(\phi) = -\mathcal{R}(-\phi). \tag{15}$$

Thus, in spatially symmetric systems, comprising inelastic interactions with an effective bias dependency, odd conductance terms acquire even symmetry with respect to the magnetic field, $\mathcal{D}(\phi) = \mathcal{D}(-\phi)$, as noted experimentally,[6,8] while even conductance terms, constructing $\mathcal{R}$, are odd with respect to ϕ. Next we show that these observations do not generally hold when a spatial asymmetry is introduced, by coupling the scattering centers unevenly to the leads.

IV. DOUBLE-DOT INTERFEROMETER

We numerically simulate an AB device with a quantum dot located at each arm of the interferometer. This setup can be prepared using two-dimensional electron gas formed in semiconductor heterostructures as in Refs. 3,4,6–8. The Hamiltonian includes the terms

$$H = H_S + \sum_{\nu=L,R,P} H_\nu + \sum_{\nu=L,R} H_{S,\nu} + H_{S,P}, \tag{16}$$

where the subsystem Hamiltonian includes two uncoupled electronic states, and the three reservoirs (metals) are composed of a collection of noninteracting electrons:

$$H_S = \sum_{n=1,2} \epsilon_n a_n^\dagger a_n, \quad H_\nu = \sum_{j\in\nu} \epsilon_j a_j^\dagger a_j. \tag{17}$$

Here $a_j^\dagger$ (a_j) are fermionic creation (annihilation) operators of electrons with momentum j and energy ϵ_j. $a_n^\dagger$ and a_n are the respective operators for the dots. The subsystem-bath coupling terms are given by

$$H_{S,L} + H_{S,R} = \sum_{n,l} v_{n,l} a_n^\dagger a_l e^{i\phi_n^L} + \sum_{n,r} v_{n,r} a_r^\dagger a_n e^{i\phi_n^R} + \text{H.c.},$$

and we assume that only dot 1 is coupled to the probe $H_{S,P} = \sum_p v_{1,p} a_1^\dagger a_p + \text{H.c.}$ Here $v_{n,j}$ is the tunneling strength of an electron from the dot n to the $j \in \nu$ bath state. Below we assume that this parameter does not depend on the dot index. ϕ_n^L and ϕ_n^R are the AB phase factors, acquired by electron waves in a magnetic field perpendicular to the device plane. These phases are constrained to satisfy $\phi_1^L - \phi_2^L + \phi_1^R - \phi_2^R = \phi$. In what follows we adopt the gauge $\phi_1^L - \phi_2^L = \phi_1^R - \phi_2^R = \phi/2$. We voltage bias the system, $\Delta\mu \equiv \mu_L - \mu_R \geqslant 0$, in a symmetric manner, $\mu_L = -\mu_R$. However, the dot energies may be placed away from the so-called symmetric point at which $\mu_L - \epsilon_n = \epsilon_n - \mu_R$ using a gate voltage. Our model does not include interacting particles, and thus its steady-state characteristics can be readily obtained using the nonequilibrium Green's-function approach.[19] Transient effects were recently explored in Refs. 20,21. In terms of the Green's function, the transmission coefficient is defined as $\mathcal{T}_{\nu,\xi} = \text{Tr}[\Gamma^\nu G^+ \Gamma^\xi G^-]$; the trace is performed over the states of the subsystem. Given our Hamiltonian, the matrix G^+ ($G^- = [G^+]^\dagger$) takes the form[22]

$$G^+ = \begin{bmatrix} \epsilon - \epsilon_1 + \frac{i(\gamma_L+\gamma_R+\gamma_P)}{2} & \frac{i\gamma_L}{2}e^{i\phi/2} + \frac{i\gamma_R}{2}e^{-i\phi/2} \\ \frac{i\gamma_L}{2}e^{-i\phi/2} + \frac{i\gamma_R}{2}e^{i\phi/2} & \epsilon - \epsilon_2 + \frac{i(\gamma_L+\gamma_R)}{2} \end{bmatrix}^{-1},$$

with the hybridization matrices

$$\Gamma^L = \gamma_L \begin{bmatrix} 1 & e^{i\phi/2} \\ e^{-i\phi/2} & 1 \end{bmatrix}, \quad \Gamma^R = \gamma_R \begin{bmatrix} 1 & e^{-i\phi/2} \\ e^{i\phi/2} & 1 \end{bmatrix}, \quad \Gamma^P = \gamma_P \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \tag{18}$$

The coupling energy between the dots and leads is given by $\gamma_\nu(\epsilon) = 2\pi \sum_{j\in\nu} |v_{n,j}|^2 \delta(\epsilon - \epsilon_j)$. In our calculations we take the wide-band limit and assume γ_ν to be energy independent parameters. The symmetry relation exposed here is not limited to this case. It is interesting to note that the transmission functions are not necessarily even in ϕ, even when Onsager symmetry is maintained. We exemplify this by considering a geometrically symmetric system with $\epsilon_d \equiv \epsilon_1 = \epsilon_2$ and $\frac{\gamma}{2} \equiv \gamma_L = \gamma_R$. The transmission functions reduce to

$$\mathcal{T}_{L,R}(\epsilon,\phi) = \mathcal{T}_{R,L}(\epsilon,-\phi) = \frac{4\gamma^2}{\Delta(\epsilon,\phi)}\left[4(\epsilon-\epsilon_d)^2\cos^2\frac{\phi}{2} + \frac{\gamma_P^2}{4} + \gamma_P(\epsilon_d - \epsilon)\sin\phi\right],$$

$$\mathcal{T}_{L,P}(\epsilon,\phi) = \mathcal{T}_{P,L}(\epsilon,-\phi) = \mathcal{T}_{P,R}(\epsilon,\phi) = \frac{4\gamma\gamma_P}{\Delta(\epsilon,\phi)}\left[2(\epsilon-\epsilon_d)^2 + \frac{\gamma^2}{2}\sin^2\frac{\phi}{2} + \gamma(\epsilon-\epsilon_d)\sin\phi\right].$$

The denominator $\Delta(\epsilon,\phi)$ is an even function of phase. When the probe dephases the system, we substitute these expressions into Eq. (10) and resolve the dephasing (D) probe distribution[22,23] $f_P^D(\epsilon,\phi) = \frac{f_L(\epsilon)+f_R(\epsilon)}{2} + \frac{\gamma(\epsilon-\epsilon_d)\sin\phi}{4[(\epsilon-\epsilon_d)^2+\omega_0^2]}[f_L(\epsilon) - f_R(\epsilon)]$ with

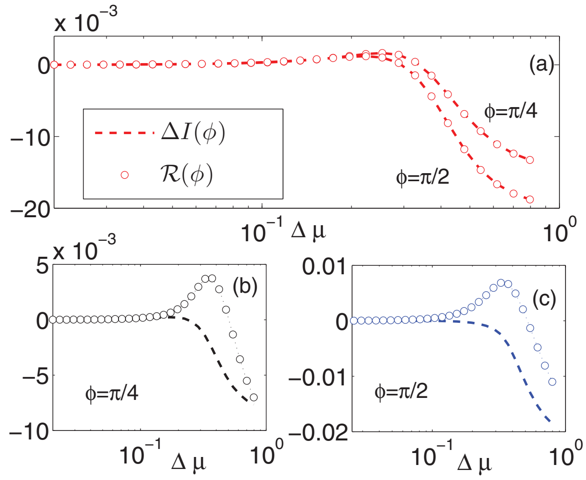

FIG. 2. (Color online) (a) Spatially symmetric junctions with $\gamma_L = \gamma_R = 0.05$ and $\mathcal{R} = \Delta I$. (b), (c) Spatial asymmetry is introduced, with $\gamma_L = 0.05$, $\gamma_R = 0.2$, resulting in $\mathcal{R} \neq \Delta I$. We adopt a voltage probe mimicking inelastic effects with $\gamma_P = 0.1$, $\epsilon_1 = \epsilon_2 = 0.15$, and inverse temperature $1/T_\nu = 50$. In all plots $\mathcal{R}$ is represented by ○ and ΔI is represented by dashed lines.

$\omega_0 = \frac{\gamma}{2}\sin\frac{\phi}{2}$. The nonequilibrium term in this distribution is *odd* in the magnetic flux. Similarly, when a voltage probe (V) is implemented, analytic results can be obtained in the linear-response regime upon solving Eq. (13):

$$\mu_P^V(\phi) = \Delta\mu \sin\phi \frac{\int d\epsilon \frac{\partial f_a}{\partial \epsilon} \frac{\gamma(\epsilon-\epsilon_d)}{\Delta(\epsilon,\phi)}}{\int d\epsilon \frac{\partial f_a}{\partial \epsilon} \frac{2(\epsilon-\epsilon_d)^2 + \frac{1}{2}\gamma^2 \sin^2\frac{\phi}{2}}{\Delta(\epsilon,\phi)}}. \tag{19}$$

Here f_a stands for the equilibrium (zero bias) Fermi-Dirac function. This chemical potential is an *odd* function of the magnetic flux, though phase rigidity is maintained in the linear-response regime.

We adopt the model Eq. (16) and implement both elastic and inelastic effects with a voltage probe, by solving the probe condition Eq. (13) numerically and iteratively[18] to obtain μ_P beyond the linear-response result, Eq. (19). We have verified that when convergence is reached the probe current is negligible, $|I_P/I_L| < 10^{-12}$. In Fig. 2 we show that $\mathcal{R} = \Delta I$ if spatial symmetry is maintained and that this relation is violated when $\gamma_L \neq \gamma_R$. Note that phase rigidity, $\Delta I = 0$, persists in the linear-response regime in all cases.[24] In Fig. 3 we display the chemical potential of the voltage probe, reflecting the interior potential of the conductor, and point out that (i) nonlinearities in the probe potential unfold as deviations of the Onsager symmetry, $\Delta I \neq 0$ (see Fig. 2), and (ii) $\mu_P(\phi)$ does not obey a magnetic-field symmetry beyond linear response. This is particularly interesting in the case of a geometrically symmetric setup, where deviations from phase rigidity do follow a certain symmetry [Eq. (15)].

In Fig. 4 we further extract the sum of odd conductance terms and confirm that it strictly satisfies $\mathcal{D}(\phi) = \mathcal{D}(-\phi)$ in spatially symmetric situations. While we noted that the symmetry of $\mathcal{R}$ is feasibly broken with small spatial asymmetry, the symmetry of $\mathcal{D}$ is more robust and its deviations are an order of magnitude smaller, in support of experimental observations.[6] Our conclusions are intact when an "up-down" asymmetry is implemented in the form $\epsilon_1 \neq \epsilon_2$ [see Figs. 4(c) and 4(d)].[25]

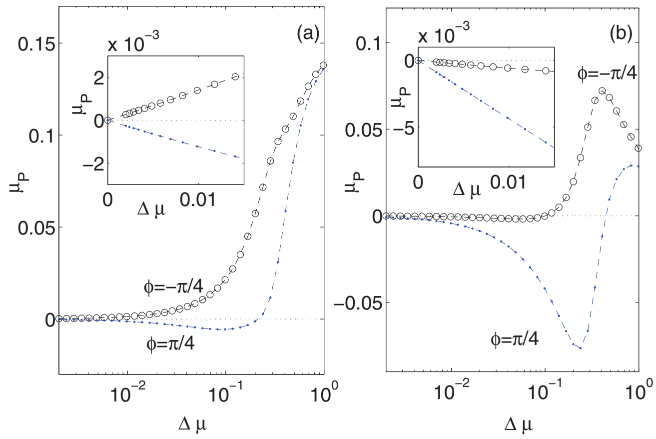

FIG. 3. (Color online) Probe chemical potential (a) in spatially symmetric junctions $\gamma_L = \gamma_R = 0.05$ and (b) when spatial asymmetry is introduced, $\gamma_L = 0.05$ and $\gamma_R = 0.2$. In all plots $\phi = \pi/4$ (dot) and $\phi = -\pi/4$ (○), the light dotted line presents the symmetry line, and the insets zoom on the main plots. Other parameters are the same as in Fig. 2

V. SUMMARY

We presented an analytical and numerical study of nonlinear transport properties of AB rings susceptible to elastic dephasing and inelastic effects by adopting the Büttiker probe method. We proved that $\mathcal{D}(\phi) = \mathcal{D}(-\phi)$ and $\mathcal{R}(\phi) = -\mathcal{R}(-\phi)$ for spatially symmetric junctions. This result is nontrivial since far from equilibrium the voltage probe parameters, introducing many-body effects, are nonlinear in the applied bias and are neither even nor odd in the magnetic flux. We also demonstrated the breakdown of these symmetries when the junction has a left-right asymmetry, in the presence of inelastic effects. It is of interest to verify these results adopting a microscopic model with genuine many-body interactions, modeling a quantum point contact[26,27] or an

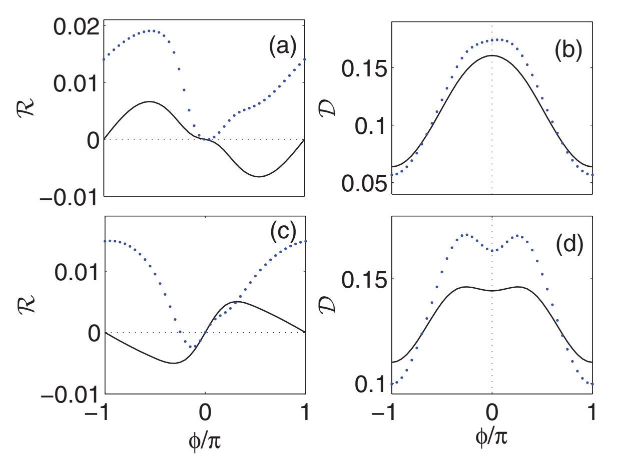

FIG. 4. (Color online) (a) Even ($\mathcal{R}$) and (b) odd ($\mathcal{D}$) conductance terms in spatially symmetric junctions, $\gamma_L = \gamma_R = 0.05$ (full) and with an asymmetry $\gamma_L = 0.05 \neq \gamma_R = 0.2$ (dotted) for $\epsilon_1 = \epsilon_2 = 0.15$. (c), (d) Same as above, only with nondegenerate levels $\epsilon_1 = 0.1$ and $\epsilon_2 = 0.2$. Parameters are the same as in Fig. 2, $\Delta\mu = 0.4$. Light dotted lines present the symmetry lines.

equilibrated phonon bath, by extending recent works[28] to the nonlinear regime. It is of interest to use the present formalism and study the thermoelectric effect, generalizing recent studies[29–31] to the far-from-equilibrium regime.

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