Full counting statistics in quantum thermodynamics

Absorption refrigerator at strong coupling

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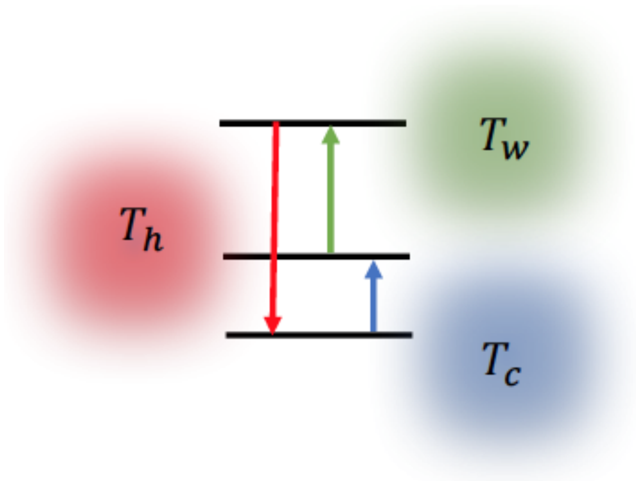
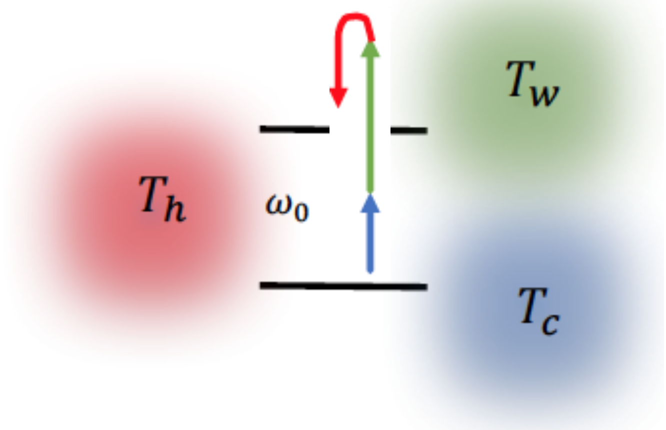




UNIVERSITY OF TORONTO

Publications

Method

Full counting statistics for energy exchange at the nanoscale: Unified description of additive and non-additive couplings

H. M. Friedman, B. K. Agarwalla, DS, in preparation

Application

Qubit absorption refrigerator at strong coupling

A. Mu, B. K. Agarwalla, G. Schaller, DS, arXiv:1709.02835

Theory

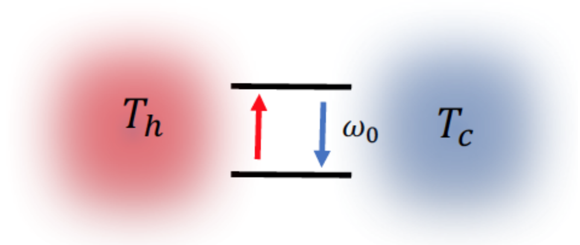
Heat transport in the strong coupling limit

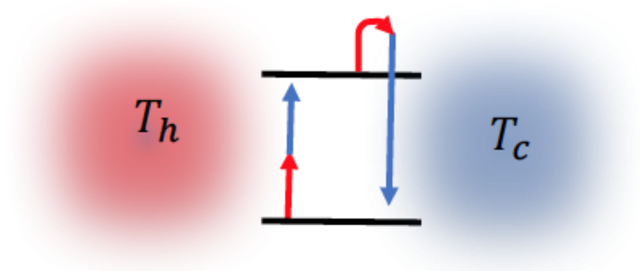
DS, PRE 90, 012148 (2014)

N. Boudjada and DS JPC A, 118 (47), 11323 (2014)

L. Nicolin and DS JCP 135, 164106 (2011)

DS, A. Nitzan, PRL. 94, 034301 (2005)





Publications

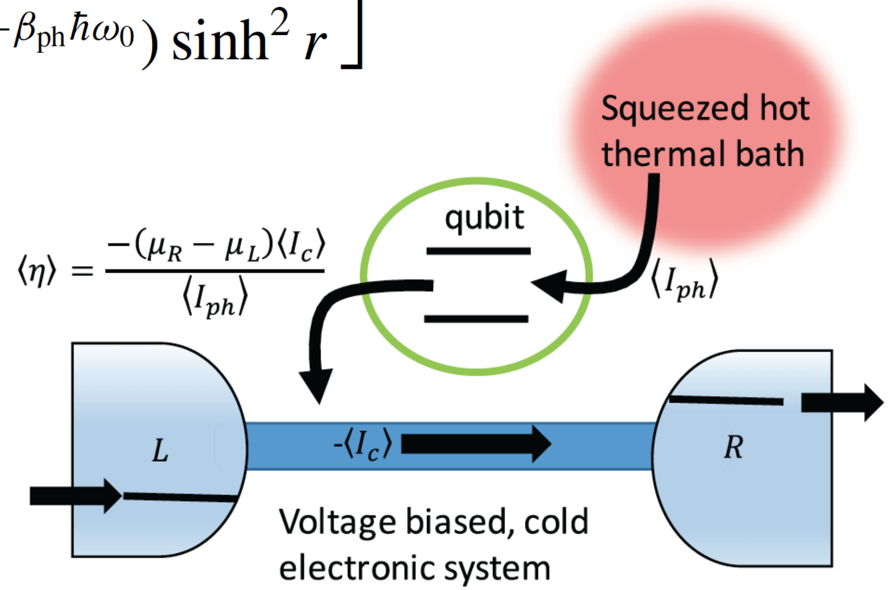
Heat engine with a squeezed bath:

Quantum efficiency bound for continuous heat engines coupled to noncanonical reservoirs

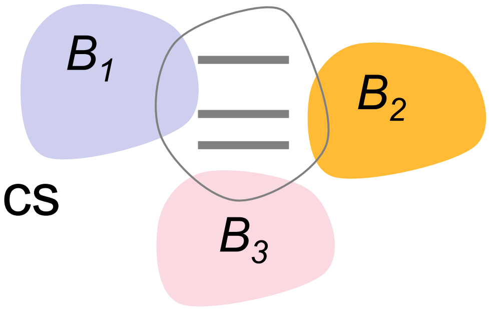
B. K. Agarwalla, J.-H. Jiang, DS, PRB 96, 104304 (2017)

$$\langle \eta \rangle \leqslant 1 - \frac{T_{\rm el}}{T_{\rm ph}} + \frac{1}{\beta_{\rm el}\hbar\omega_0} \ln\left[\frac{1+(1+e^{\beta_{\rm ph}\hbar\omega_0})\sinh^2 r}{1+(1+e^{-\beta_{\rm ph}\hbar\omega_0})\sinh^2 r}\right]$$

$$\langle \eta \rangle \leqslant 1 - \frac{T_{\rm el}}{T_{\rm ph}(1+2\sinh^2 r)}$$



Outline

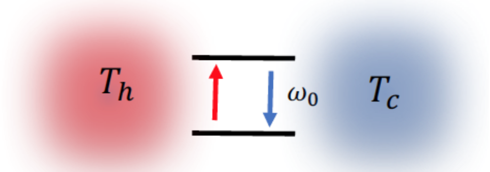


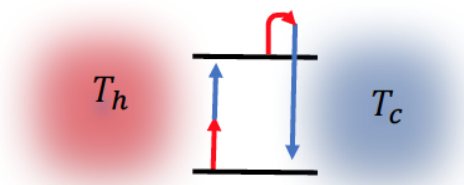
1. **Introduction**: nonequilibrium statistical mechanics (fluctuation theorem, second law, cumulants)

2. **Formalism:** Full counting statistics for energy exchange

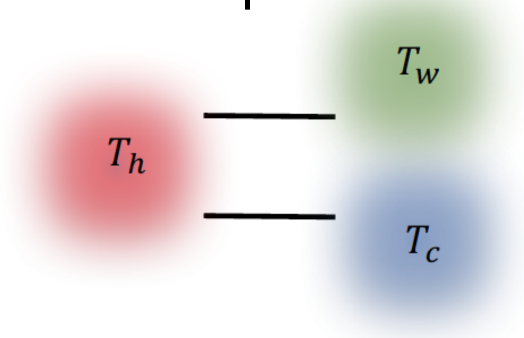
$$\hat{V}_N = \sum_{\nu=1}^{N} \hat{B}_\nu \qquad \hat{V}_N = \Pi_{\nu=1}^{N} \hat{B}_\nu$$

3. **Two terminal** – nonlinear energy transport



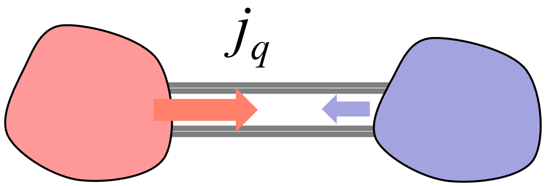


4. **Three-terminal** - quantum absorption refrigerator



Exchange fluctuation theorems:
Microscopic statement of the second law

$$\ln\left[\frac{P_t(+Q)}{P_t(-Q)}\right] = (\beta_R - \beta_L)Q$$



$$\ln\left[\frac{P_t(+S)}{P_t(-S)}\right] = S$$

Characteristic function

$$\mathcal{Z}(\lambda) = \int dS e^{i\lambda S} P(S)$$

$$P(S) = e^S P(-S) \;\leftrightarrow\; \mathcal{Z}(\lambda) = \mathcal{Z}(-\lambda + i)$$

Gallavotti – Cohen fluctuation symmetry

$$1 = \langle e^{-S} \rangle \quad \xrightarrow{e^{\langle x \rangle} \leq \langle e^x \rangle} \quad \langle S \rangle \geq 0$$

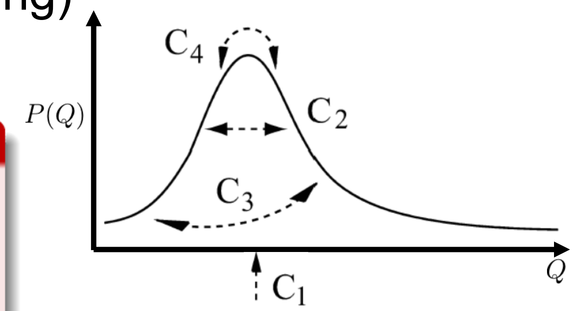
Second law of thermodynamics!

- Familiar inequalities of macroscopic thermodynamics are now equalities!
- Thermodynamic laws are recovered at the level of the ensemble average

M. Esposito, U. Harbola, S. Mukamel, Rev. Mod. Phys. 81, 1665 (2009).
M. Campisi, P. Hanggi, P. Talkner, Rev. Mod. Phys. 83, 771 (2011).

What is Full-Counting Statistics (FCS)?

- **Full probability distribution** of transferred charge, heat, in a given time interval
- Origin of FCS - Quantum optics (Photon counting)



Cumulants

- 1st cumulant – mean: $C_1 = \langle Q \rangle$.
- 2nd cumulant – Variance : $C_2 \equiv \langle\langle Q^2 \rangle\rangle = \langle (Q - \langle Q \rangle)^2 \rangle$.
- 3rd cumulant – Skewness: $C_3 = \langle\langle Q^3 \rangle\rangle = \langle (Q - \langle Q \rangle)^3 \rangle$.

- Instead of the distribution we often look at the characteristic function

$$\mathcal{Z}(\lambda) = \int dQ\, P(Q)\, e^{i\lambda Q}$$

$$\langle\langle Q^m \rangle\rangle = \frac{\partial^m \ln \mathcal{Z}(\lambda)}{\partial (i\lambda)^m}\Big|_{\lambda=0}$$

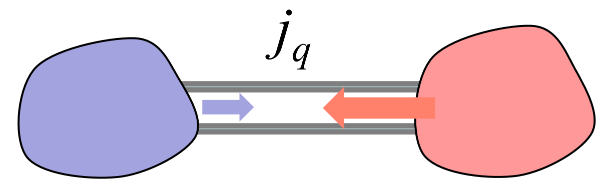
Cumulant generating function

Full Counting Statistics: Two-time measurement

How do we construct the probability distribution?

- Work, heat transferred charge are not observables
- They characterize processes from initial time to final time

Total heat flow $Q_L(t, t_0) = H_L(t_0) - H_L^H(t)$



$H = H_S + H_L + H_R + H_{SL} + H_{SR}$

Two-time Measurement

- Measure energy at the initial time

  Collapse of wave function ϕ_i with energy eigenvalue ϵ_i

- Time evolution upto the final measurement time

- Measure at the final time

  Collapse of wave function ϕ_j with energy eigenvalue ϵ_j

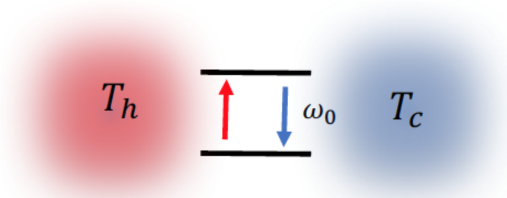
Probability distribution $P_{i \to j}(Q) = \delta(Q - (\epsilon_i - \epsilon_j)|\langle \phi_j | U(t, t_0) | \phi_i \rangle|^2$

Characteristic function for heat $\boxed{\mathcal{Z}(\lambda) = \langle e^{i\lambda H_L(t_0)} e^{-i\lambda H_L^H(t)} \rangle}$

Heat exchange

$$\hat{H} = \hat{H}_S + \sum_{\nu=1}^{N} \hat{H}_\nu + \hat{S} \otimes \hat{V}_N$$

energy change at the ν bath



$$\hat{Q}_\nu(t, t_0 = 0) = \hat{H}_\nu(0) - \hat{H}_\nu(t)$$

$$\mathcal{Z}(\lambda = \{\lambda_\nu\}, t) \equiv \mathrm{Tr}\left[e^{i\sum_\nu \lambda_\nu \hat{H}_\nu(0)} e^{-i\sum_\nu \lambda_\nu \hat{H}_\nu(t)} \rho(0)\right]$$

$$\mathcal{G}(\lambda) \equiv \lim_{t\to\infty} \frac{1}{t} \ln \mathcal{Z}(\lambda)$$

cumulant generating function

$$\boxed{\mathcal{Z}(\{\lambda_\nu\}, t) \equiv \mathrm{Tr}\left[\hat{U}_{-\lambda}(t)\rho(0)\hat{U}_\lambda^\dagger(t)\right] \equiv \mathrm{Tr}\left[\rho^\lambda(t)\right]}$$

$$\hat{U}_{-\lambda}(t) \equiv e^{-i\sum_\nu \lambda_\nu \hat{H}_\nu/2} \hat{U}(t) e^{i\sum_\nu \lambda_\nu \hat{H}_\nu/2}$$

$$\hat{U}_\lambda^\dagger(t) \equiv e^{i\sum_\nu \lambda_\nu \hat{H}_\nu/2} \hat{U}^\dagger(t) e^{-i\sum_\nu \lambda_\nu \hat{H}_\nu/2}$$

Redfield Equation (Born Markov approximation)

$$\sigma^\lambda(t) \equiv \mathrm{Tr}_B\left[\rho^\lambda(t)\right]$$

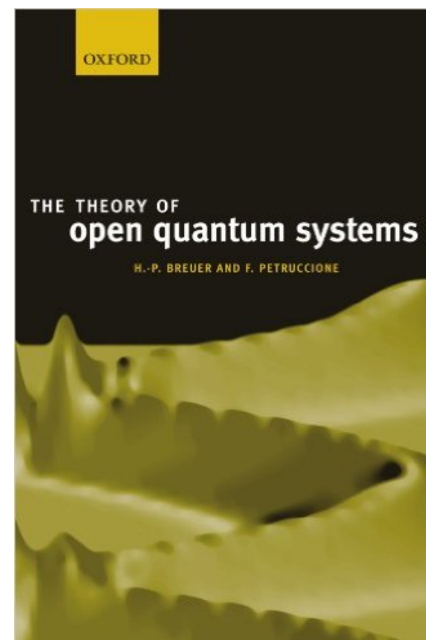
$$\dot{\sigma}^\lambda_{nm}(t) = -i[\hat{H}_S, \sigma(t)]_{nm} + \sum_{j,k}\Big[- \sigma^\lambda_{km}(t) R^{(+)}_{N_{nj,jk}}(E_{k,j}) - \sigma^\lambda_{nj}(t) R^{(-)}_{N_{km,jk}}(E_{j,k}) \\ + \sigma^\lambda_{jk}(t)\Big(R^{\lambda,-\lambda\,(-)}_{N_{km,nj}}(E_{k,m}) + R^{\lambda,-\lambda\,(+)}_{N_{km,nj}}(E_{j,n})\Big)\Big]$$

$$\rho(0) = \sigma(0) \otimes \rho_B(0)$$

$$\rho_B(0) = \prod_{\nu=1}^{N} \rho_\nu \qquad \rho_\nu(0) = \exp[-\beta_\nu \hat{H}_\nu]/Z_\nu$$

$$M^{\lambda,\lambda'}_{N_{ab,cd}}(s) \equiv \langle \hat{V}^\lambda_N(s)\hat{V}^{\lambda'}_N(0)\rangle\, S_{a,b}S_{c,d}$$

$$R^{\lambda,\lambda'\,(+)}_{N_{ab,cd}}(\omega) \equiv \int_0^\infty e^{i\omega s} M^{\lambda,\lambda'}_{N_{ab,cd}}(s)\, ds$$



Secular Approximation

$$
\begin{aligned}
\dot{p}_n^{\lambda}(t) = & -\; p_n^{\lambda}(t) \sum_j \int_{-\infty}^{\infty} e^{i\,E_{n,j}\,s} M_{N_{nj,jn}}(s)\, ds \\
& + \sum_j p_j^{\lambda}(t) \int_{-\infty}^{\infty} e^{i\,E_{j,n}\,s} M^{\lambda,-\lambda}_{N_{nj,jn}}(s)\, ds
\end{aligned}
$$

solve in steady state

$$\mathcal{Z}(\lambda) = \langle 1 | p_\lambda(t \to \infty) \rangle$$

$$\langle J \rangle = \lim_{t\to\infty} \frac{1}{t} \frac{\partial \ln \mathcal{Z}(\lambda)}{\partial i\lambda} \Big|_{\lambda=0}$$

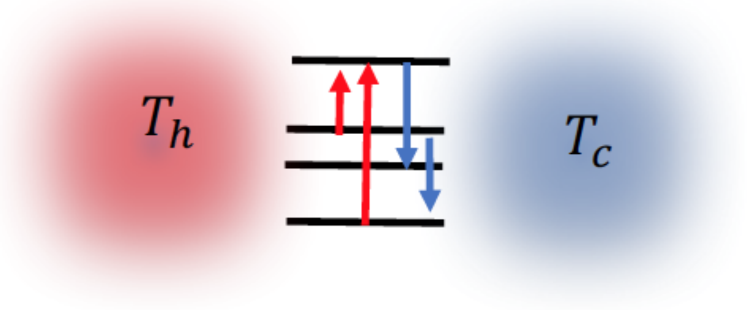
$$\hat{V}_N = \sum_{\nu=1}^{N} \hat{B}_\nu$$

$$\hat{V}_N = \Pi_{\nu=1}^{N} \hat{B}_\nu$$

$$|\dot{p}^{\lambda}(t)\rangle = \mathcal{L}^{\lambda}\, |p^{\lambda}(t)\rangle$$

$$\langle J \rangle = \langle 1 | \frac{\partial \mathcal{L}^{\lambda}}{\partial i\lambda}\Big|_{\lambda=0} |p_{ss}\rangle$$

Additive model $\hat{V}_N = \hat{B}_L + \hat{B}_R$



Two-time correlation function in time

$$M^{\lambda,\lambda'}_{(L,R)ab,cd}(s) = \left(\langle \hat{B}_L(s)\hat{B}_L(0)\rangle + \langle \hat{B}^{\lambda}_R(s)\hat{B}^{\lambda'}_R(0)\rangle\right) S_{a,b}S_{c,d}$$

Fourier Transform (dissipation rate)

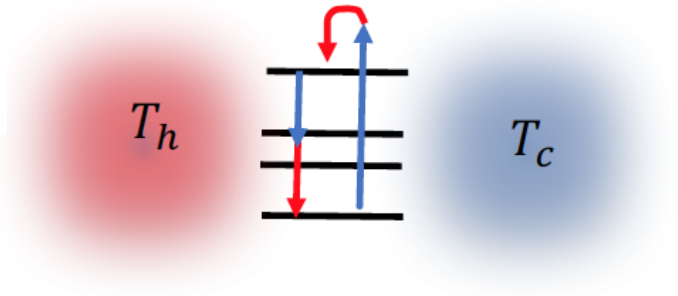
$$M^{\lambda,-\lambda}_{(L,R)_{nj;jn}}(E_{j,n}) = M_{L_{nj;jn}}(E_{j,n}) + M^{\lambda,-\lambda}_{R_{nj;jn}}(E_{j,n})$$

$$\boxed{|\dot{p}^{\lambda}(t)\rangle = \left(\mathcal{L}^{\lambda}_R + \mathcal{L}_L\right)|p^{\lambda}(t)\rangle}$$

$$\boxed{\langle J\rangle = \langle 1|\frac{\partial \mathcal{L}^{\lambda}}{\partial i\lambda}\Big|_{\lambda=0}|p_{ss}\rangle}$$

$$\langle J\rangle \;=\; \mathrm{Tr}[\hat{H}_S\mathcal{L}_R p]$$

Non-Additive model $\hat{V}_N = \hat{B}_L \otimes \hat{B}_R$



Two-time correlation function in time

$$M^{\lambda,\lambda'}_{(L,R)_{ab,cd}}(s) = \langle \hat{B}_L(s)\hat{B}_L(0)\rangle\langle \hat{B}^{\lambda}_R(s)\hat{B}^{\lambda'}_R(0)\rangle\, S_{a,b}S_{c,d}$$

Fourier Transform (dissipation rate)

$$M^{\lambda,-\lambda}_{(L,R)_{nj;jn}}(E_{j,n}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} M_{L_{nj;jn}}(E_{j,n}-\omega)M^{\lambda,-\lambda}_{R_{nj;jn}}(\omega)d\omega$$

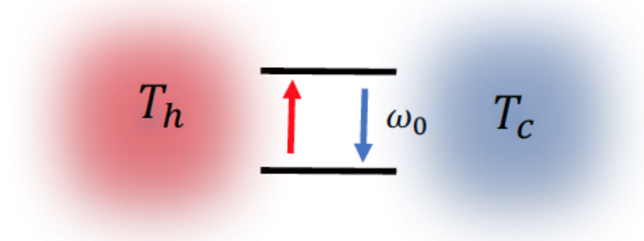
$$\boxed{\langle J\rangle = \langle 1|\frac{\partial \mathcal{L}^{\lambda}}{\partial i\lambda}\Big|_{\lambda=0}|p_{ss}\rangle}$$

$$\langle J\rangle = -\sum_{n,j} p_j \int_{-\infty}^{\infty} d\omega\omega M_{L_{nj;jn}}(E_{j,n}-\omega)M_{R_{nj;jn}}(\omega)$$

Two-state model

$$\hat{H}_S = \frac{\omega_0}{2}\hat{\sigma}_z = \frac{\omega_0}{2}\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right)$$

$$\hat{S} = \hat{\sigma}_x = |0\rangle\langle 1| + |1\rangle\langle 0| .$$



$$\mathcal{G}(\lambda) = -\frac{1}{2}(k_{0\to 1} + k_{1\to 0}) + \frac{1}{2}\sqrt{(k_{0\to 1} + k_{1\to 0})^2 - 4\left(k_{0\to 1}k_{1\to 0} - k_{0\to 1}^{\lambda}k_{1\to 0}^{\lambda}\right)}$$

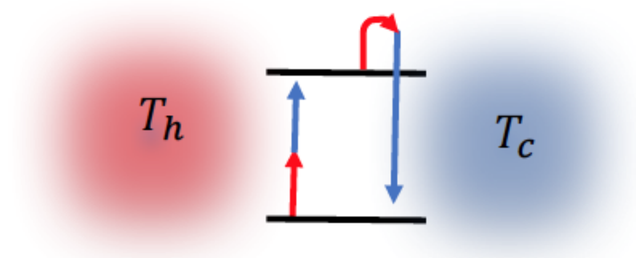
Rate constants are sum of L and R baths processes

$$\hat{V}_N = \hat{B}_L + \hat{B}_R$$

$$\boxed{\langle J\rangle = \omega_0[p_0 k_{0\to 1}^R - p_1 k_{1\to 0}^R]}$$

Two-state model

$$\hat{H}_S = \frac{\omega_0}{2}\hat{\sigma}_z = \frac{\omega_0}{2}\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right)$$
$$\hat{S} = \hat{\sigma}_x = |0\rangle\langle 1| + |1\rangle\langle 0| .$$



$$\mathcal{G}(\lambda) = -\frac{1}{2}(k_{0\to 1} + k_{1\to 0}) + \frac{1}{2}\sqrt{(k_{0\to 1} + k_{1\to 0})^2 - 4\left(k_{0\to 1}k_{1\to 0} - k^{\lambda}_{0\to 1}k^{\lambda}_{1\to 0}\right)}$$

Rate constants are **convolution** of L and R processes

$$\hat{V}_N = \hat{B}_L \otimes \hat{B}_R$$

Energy exchange is a **cooperative process**

$$\boxed{\langle J\rangle = \frac{1}{2\pi}\left[p_0\int_{-\infty}^{\infty} d\omega\omega M_L(-\omega_0+\omega)M_R(-\omega) - p_1\int_{-\infty}^{\infty} d\omega\omega M_L(\omega_0-\omega)M_R(\omega)\right].}$$

Spin-boson model

$$\hat{H} = \frac{\omega_0}{2}\sigma_z + \frac{\Delta}{2}\sigma_x + \sigma_z \sum_{\nu,j} \lambda_{j,\nu}(\hat{b}^\dagger_{j,\nu} + \hat{b}_{j,\nu}) + \sum_{\nu,j} \omega_j \hat{b}^\dagger_{j,\nu}\hat{b}_{j,\nu}$$

After the Polaron transformation

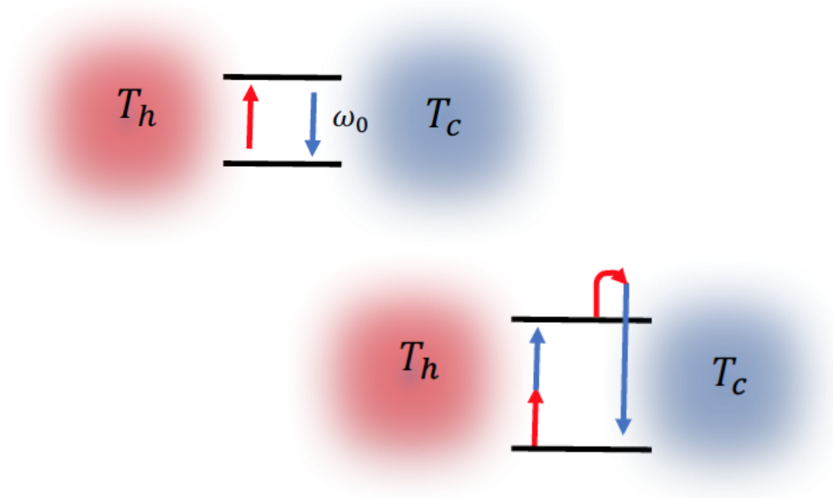
$$\hat{H}_p = \frac{\omega_0}{2}\sigma_z + \frac{\Delta}{2}\left(\sigma_+ e^{i\Omega} + \sigma_- e^{-i\Omega}\right) + \sum_{\nu,j} \omega_j \hat{b}^\dagger_{j,\nu}\hat{b}_{j,\nu}$$

$$\Omega = \sum_\nu \Omega_\nu$$

$$\Omega_\nu = 2i\sum_j \frac{\lambda_{j,\nu}}{\omega_j}(\hat{b}^\dagger_{j,\nu} - \hat{b}_{j,\nu})$$

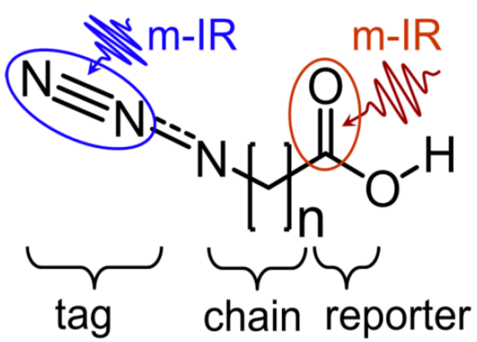
Two terminal spin boson model: nonlinear energy transport

Full Counting Statistics: cumulants, fluctuation theorem, scaling laws

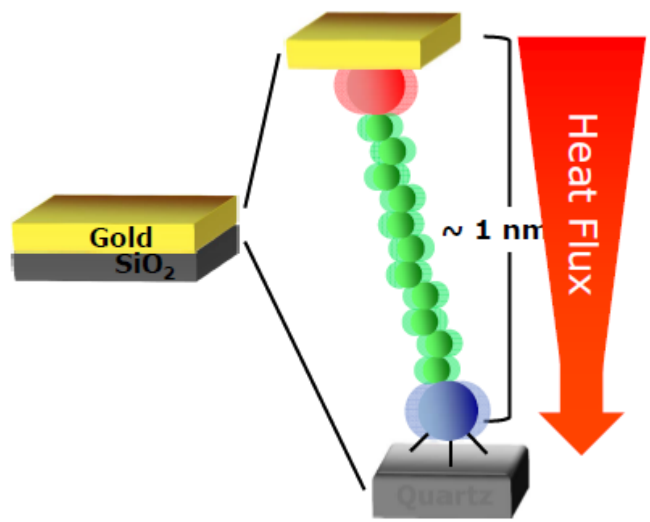


Method Development:
Approximate → exact
(QME, NEGF, path integral)

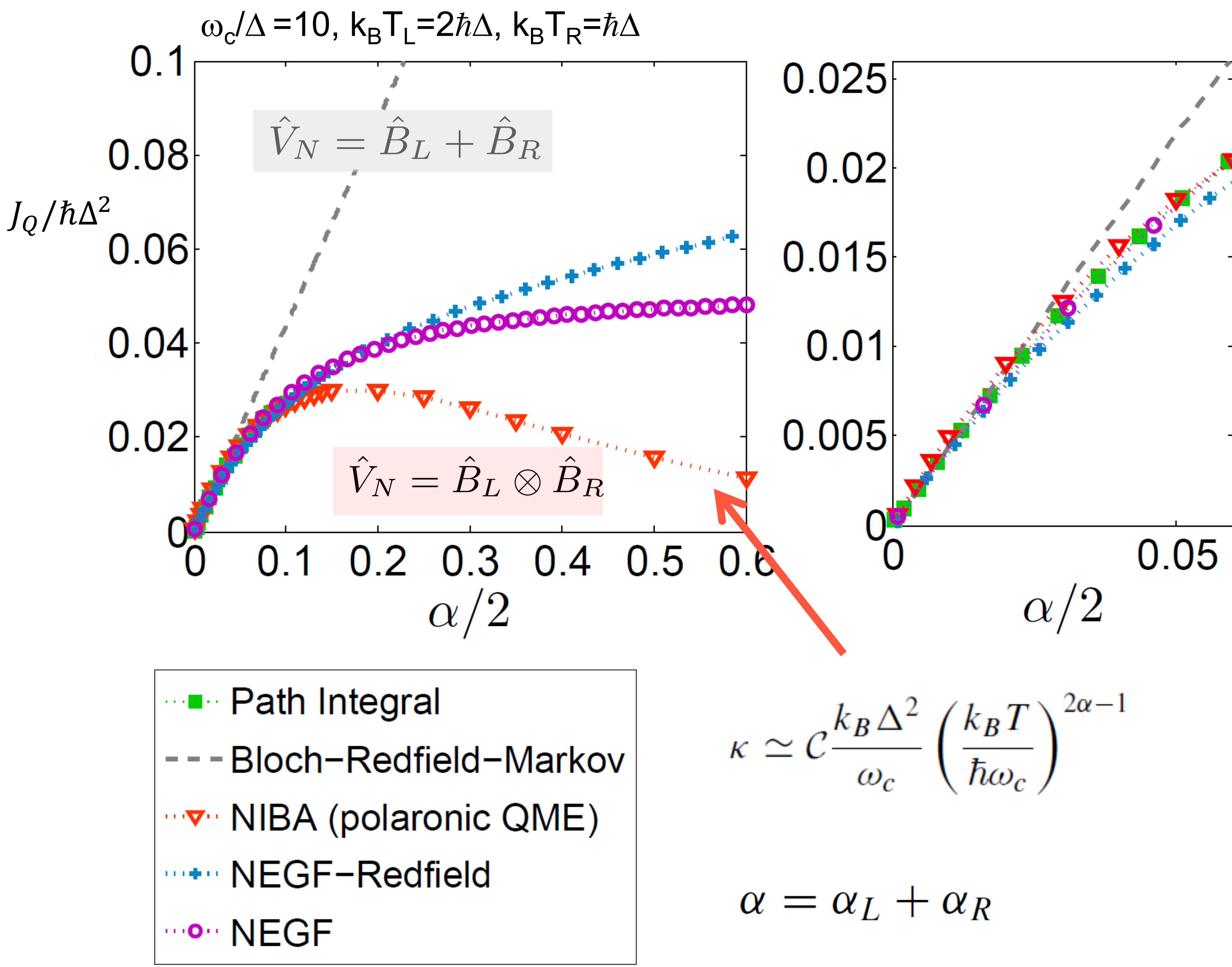
DS, PRE 90, 012148 (2014)
N. Boudjada and DS JPC A, 118 (47), 11323 (2014)
L. Nicolin and DS JCP 135, 164106 (2011)
DS, Phys. Rev. B, 73, 205415 (2006)
DS, A. Nitzan, PRL. 94, 034301 (2005)



Rubtsov (Tulane)



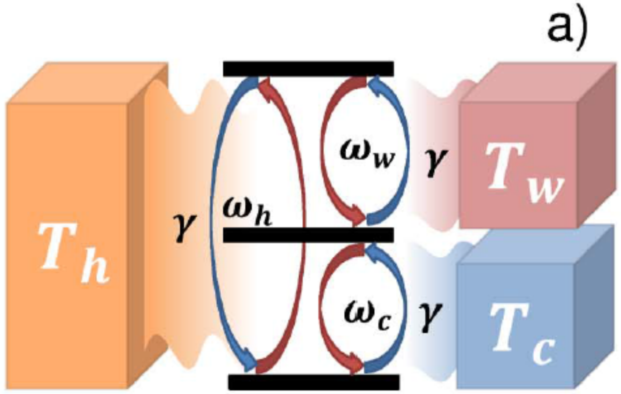
Cahill (Illinois)

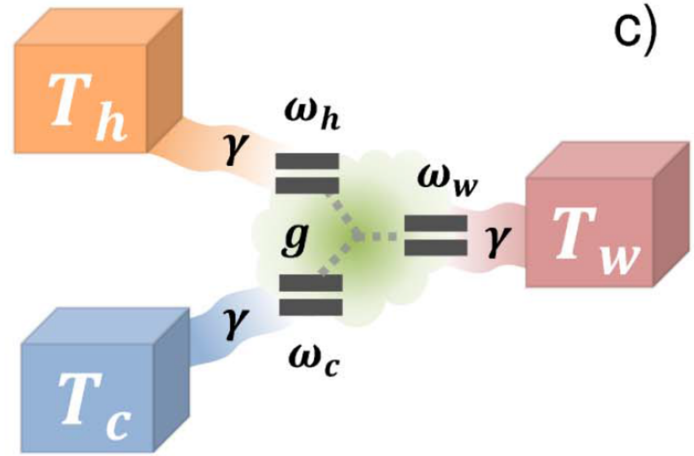


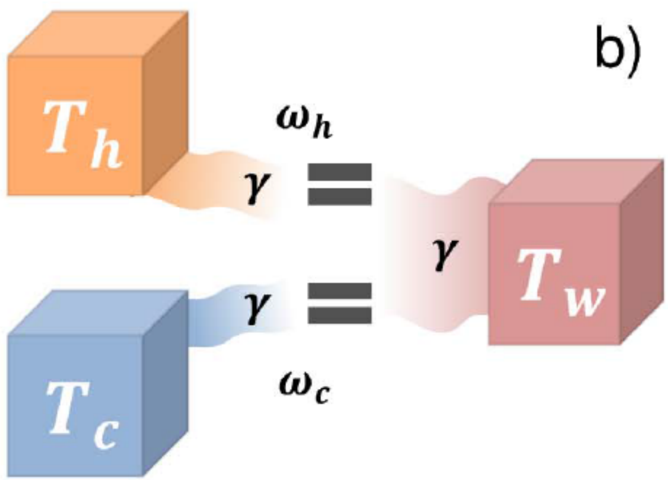
$$\kappa \simeq \mathcal{C}\frac{k_B\Delta^2}{\omega_c}\left(\frac{k_BT}{\hbar\omega_c}\right)^{2\alpha-1}$$

$$\alpha = \alpha_L + \alpha_R$$

Quantum Absorption Refrigerator







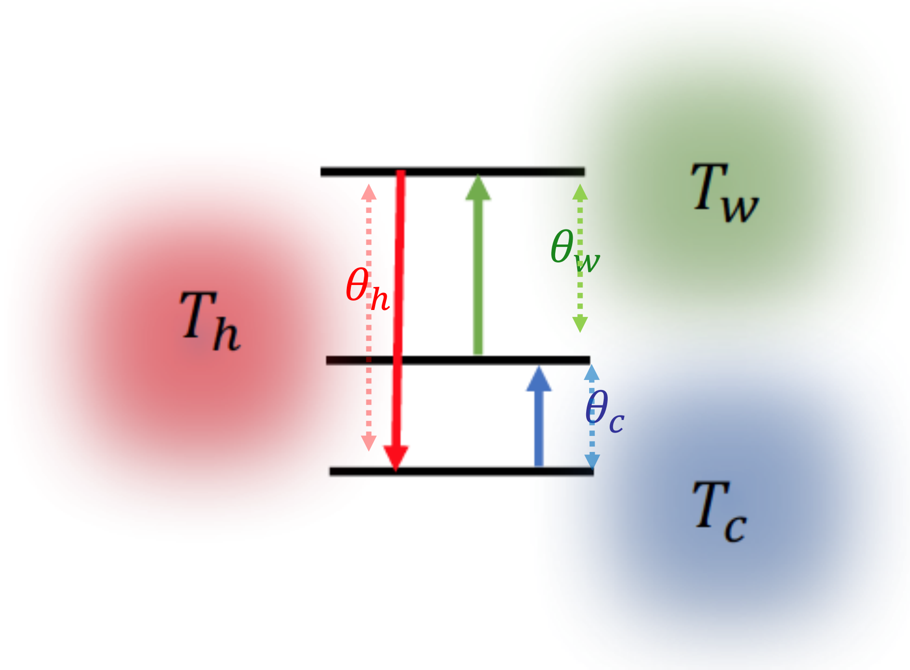
Quantum heat engines and refrigerators
R. Kosloff and A. Levy, ARPC 65 365 (2014).

Quantum enhanced absorption refrigerators,
L. A. Correa, J. P. Palao, D. Alonso, G. Adesso
Sci. Reports 4 3949 (2014).

Weak coupling to the three reservoirs
control over the system: energies, transitions, inter-system coupling.

Strong coupling to three reservoirs: filtering the baths' frequencies.

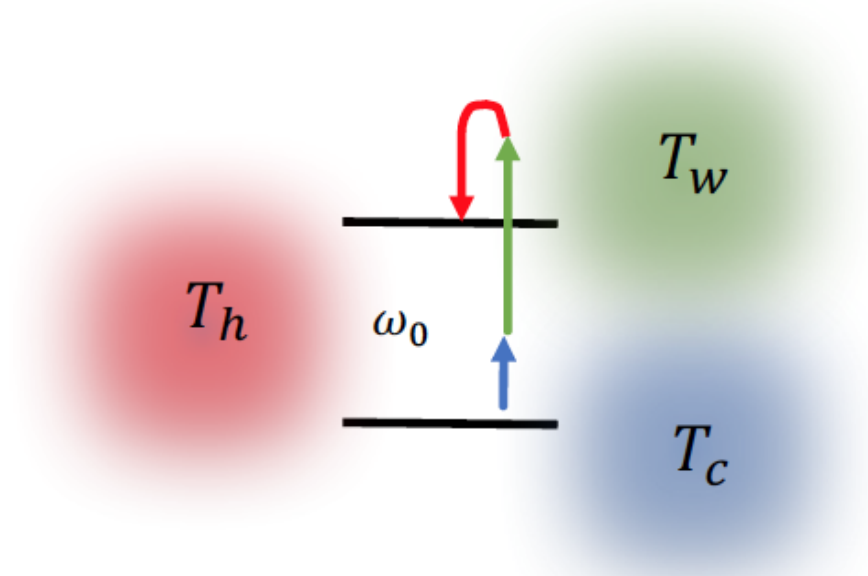
Three-level, weak coupling quantum absorption refrigerator



$$\left(\frac{T_w - T_h}{T_w - T_c}\right)\frac{T_c}{T_h} \geq \frac{\theta_c}{\theta_h}$$

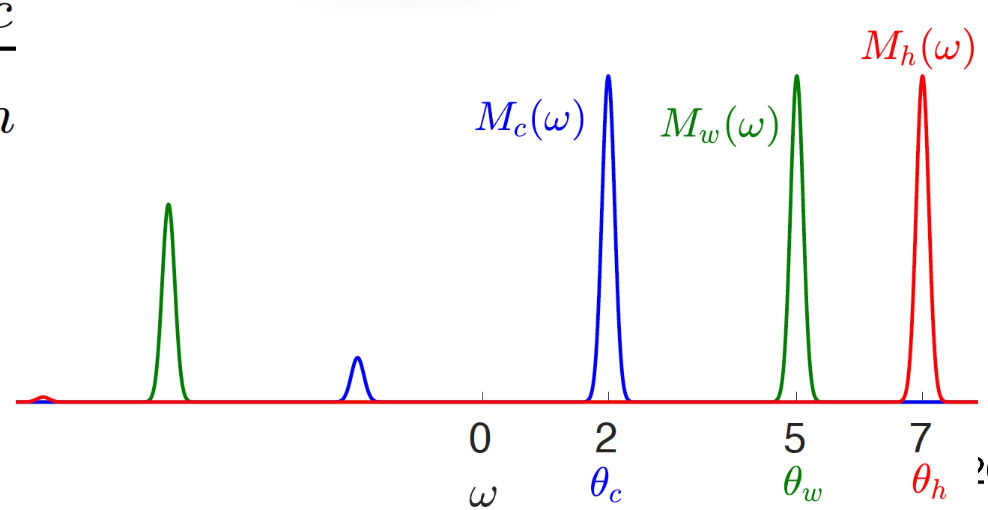
Derivation based on the weak system bath coupling model with additive dissipation

Nonequilibrium spin boson model: quantum absorption refrigerator at strong coupling



$$\left(\frac{T_w - T_h}{T_w - T_c}\right)\frac{T_c}{T_h} \geq \frac{\theta_c}{\theta_h}$$

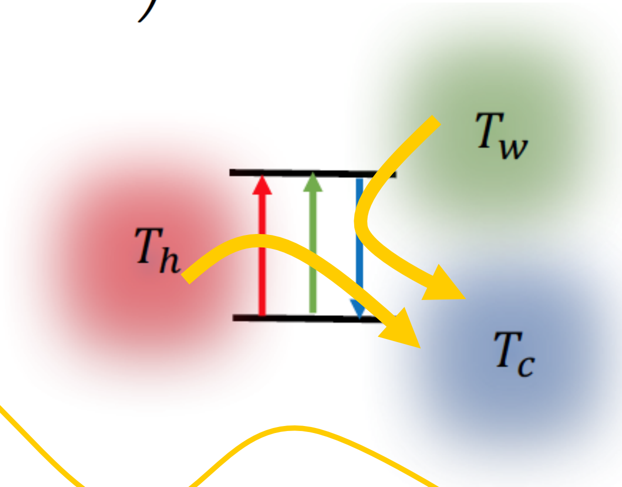
$$\theta_c + \theta_w = \theta_h$$



$$\hat{H} = \frac{\omega_0}{2}\hat{\sigma}_z + \sum_\nu \hat{H}_{B,\nu} + \frac{\hat{\sigma}_x}{2} \otimes \left(\hat{A}_c + \hat{A}_h + \hat{A}_w\right)$$

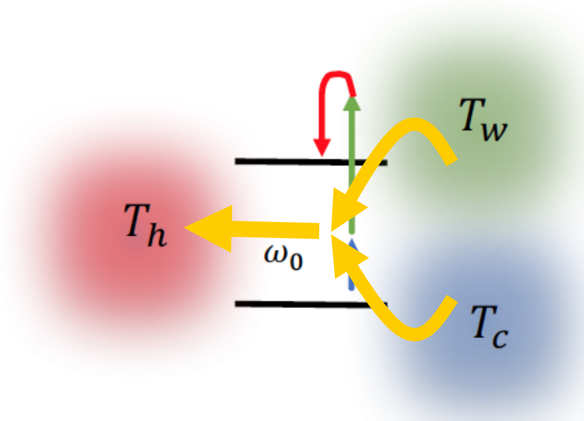
Energy current flows independently, from every hot both to cold bath.

No cooling



$$\hat{H} = \frac{\omega_0}{2}\hat{\sigma}_z + \sum_\nu \hat{H}_{B,\nu} + \gamma\frac{\hat{\sigma}_x}{2} \otimes \left(\hat{B}_c \otimes \hat{B}_h \otimes \hat{B}_w\right)$$

Energy current flows in a cooperative way. Can extract energy from the cold bath, assisted by the work reservoir, and dump into the hot bath.



$$\dot{p}_e = -M(\omega_0)p_e(t) + M(-\omega_0)p_g(t)$$

$$\begin{aligned} J_c &= -p_e \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega_1 d\omega_2 \omega_1 M_c(\omega_1) M_h(\omega_2) M_w(\omega_0 - \omega_1 - \omega_2) \\ &\quad + p_g \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega_1 d\omega_2 \omega_1 M_c(-\omega_1) M_h(-\omega_2) M_w(\omega_1 + \omega_2 - \omega_0) \end{aligned}$$

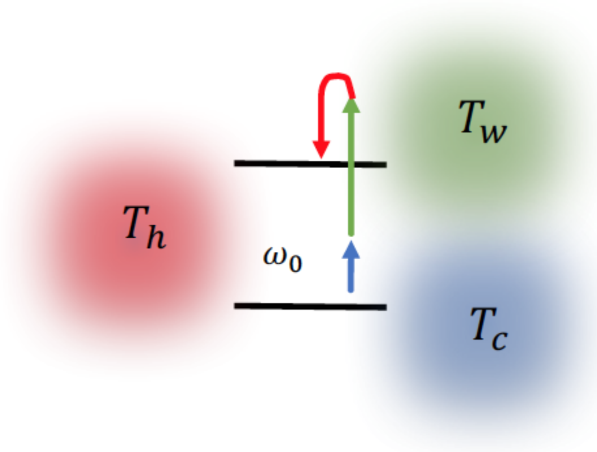
$$M_\nu(t) = \langle \hat{B}_\nu(t)\hat{B}_\nu(0)\rangle \qquad \frac{M_\nu(\omega)}{M_\nu(-\omega)} = e^{\beta_\nu \omega}$$

$$\begin{aligned} M(\omega_0) &= \left(\frac{\gamma}{2}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\omega_0 t} M_c(t) M_h(t) M_w(t) dt \\ &= \frac{1}{(2\pi)^2}\left(\frac{\gamma}{2}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M_c(\omega_1) M_h(\omega_2) M_w(\omega_0 - \omega_1 - \omega_2) d\omega_1 d\omega_2 \end{aligned}$$

Second order perturbation theory in the system-bath coupling.
Born-Markov, secular approximation

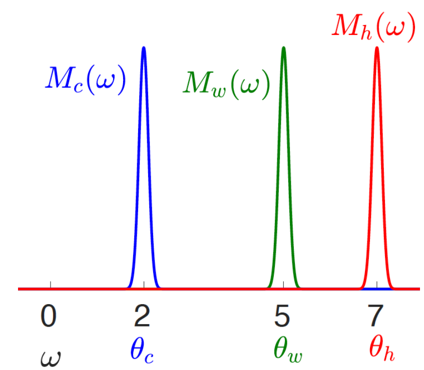
Ideal design: filter the baths

$$J_c \; = \; -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega_1 d\omega_2 \omega_1 M_c(\omega_1) M_h(\omega_2) M_w(-\omega_1 - \omega_2)$$



$$M_\nu(\omega) = \epsilon_\nu \left[\delta(\omega - \theta_\nu) + \delta(\omega + \theta_\nu) e^{+\beta_\nu \omega}\right]$$

$$\theta_c + \theta_w = \theta_h$$



cooling window

$$\left(\frac{T_w - T_h}{T_w - T_c}\right)\frac{T_c}{T_h} \geq \frac{\theta_c}{\theta_h}$$

Efficiency at maximal cooling power

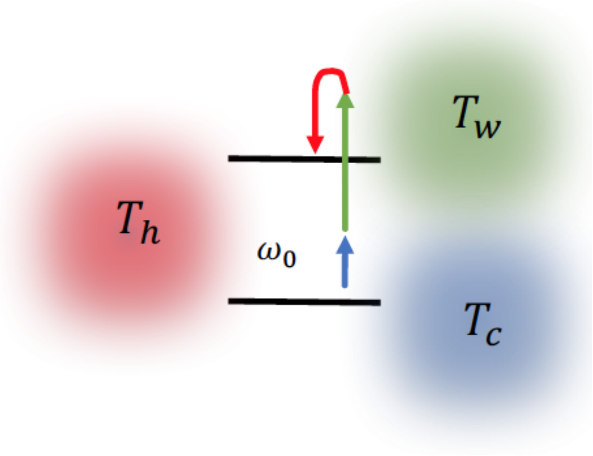
$$\frac{\partial J_c}{\partial \theta_c} = 0$$

Maximal coefficient of performance (efficiency)

$$\eta = \frac{\theta_c}{\theta_w} \leq \left(\frac{T_w - T_h}{T_h - T_c}\right)\frac{T_c}{T_w} \equiv \eta_c$$

$$\eta^* \leq \frac{1}{2}\frac{\beta_h - \beta_w}{\beta_c - \beta_h} = \frac{1}{2}\eta_c$$

Non-Ideal design



$$
\begin{aligned}
M_c(\omega) &= \epsilon_c \delta(\omega - \theta_c), \\
M_h(\omega) &= \epsilon_h \left[H(\omega - \theta_h + \delta) - H(\omega - \theta_h - \delta)\right], \\
M_w(\omega) &= \epsilon_w \left[H(\omega - \theta_w + \delta) - H(\omega - \theta_w - \delta)\right]
\end{aligned}
$$

cooling window
(non-universal)

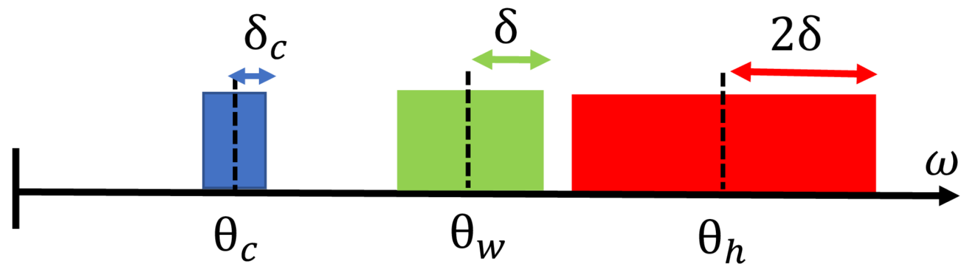
$$
\left(\frac{T_w - T_h}{T_w - T_c}\right)\frac{T_c}{T_h} \geq \frac{\theta_c}{\theta_h} - \frac{T_c T_w}{\theta_h (T_w - T_c)} \ln\left[\frac{2 + \frac{{\beta_w}^2 \delta^2}{3}}{2 + \frac{{\beta_h}^2 \delta^2}{3}}\right]
$$

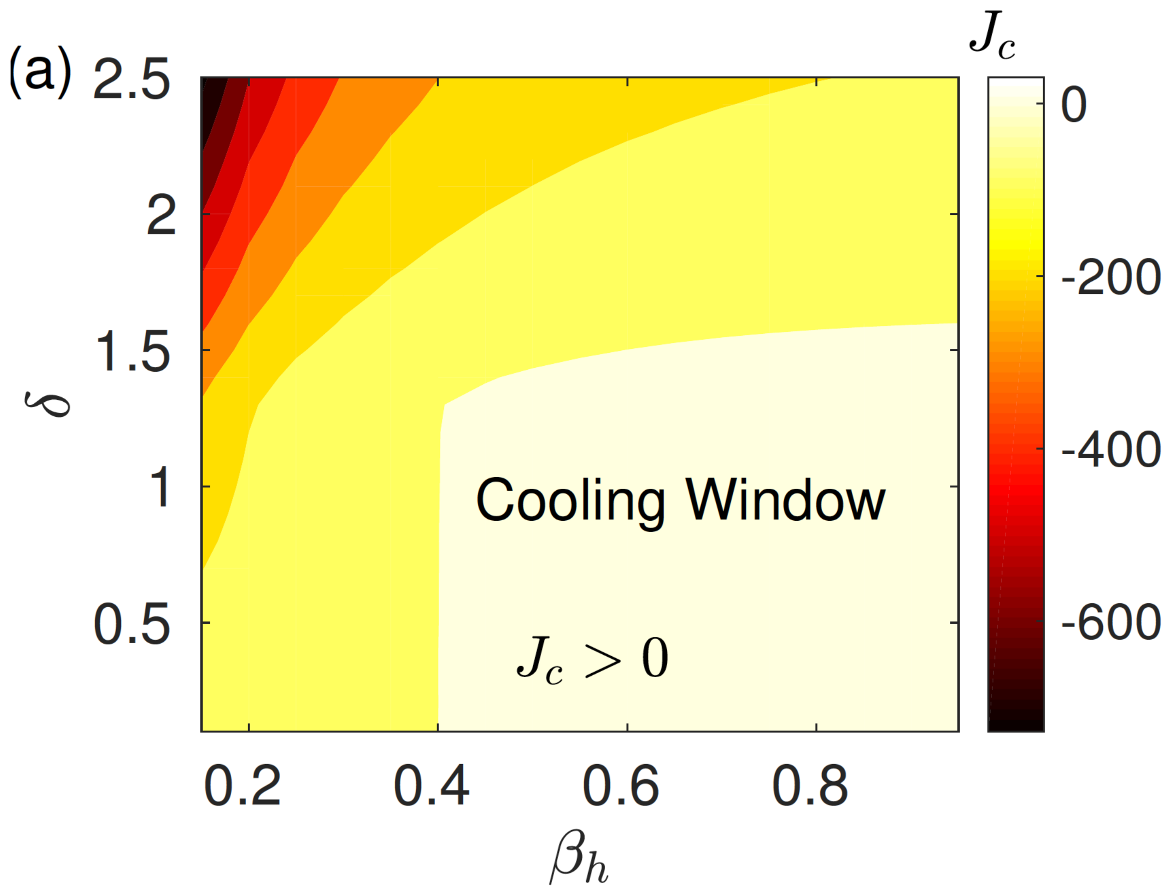
COP

$$
\eta = \frac{\theta_c e^{-\beta_c \theta_c} e^{-\beta_w \theta_w} \left[2\delta + \frac{1}{3}{\beta_w}^2 \delta^3\right] - \theta_c e^{-\beta_h \theta_h} \left[2\delta + \frac{1}{3}{\beta_h}^2 \delta^3\right]}{e^{-\beta_c \theta_c} e^{-\beta_w \theta_w} \left[2\theta_w \delta - \frac{2}{3}\beta_w \delta^3 + \frac{1}{3}\theta_w {\beta_w}^2 \delta^3\right] - e^{-\beta_h \theta_h} \left[2\theta_w \delta - \frac{2}{3}\beta_h \delta^3 + \frac{1}{3}\theta_w {\beta_h}^2 \delta^3\right]}
$$

Cooling window

$$J_c = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega_1 d\omega_2 \omega_1 M_c(\omega_1) M_h(\omega_2) M_w(-\omega_1-\omega_2)$$





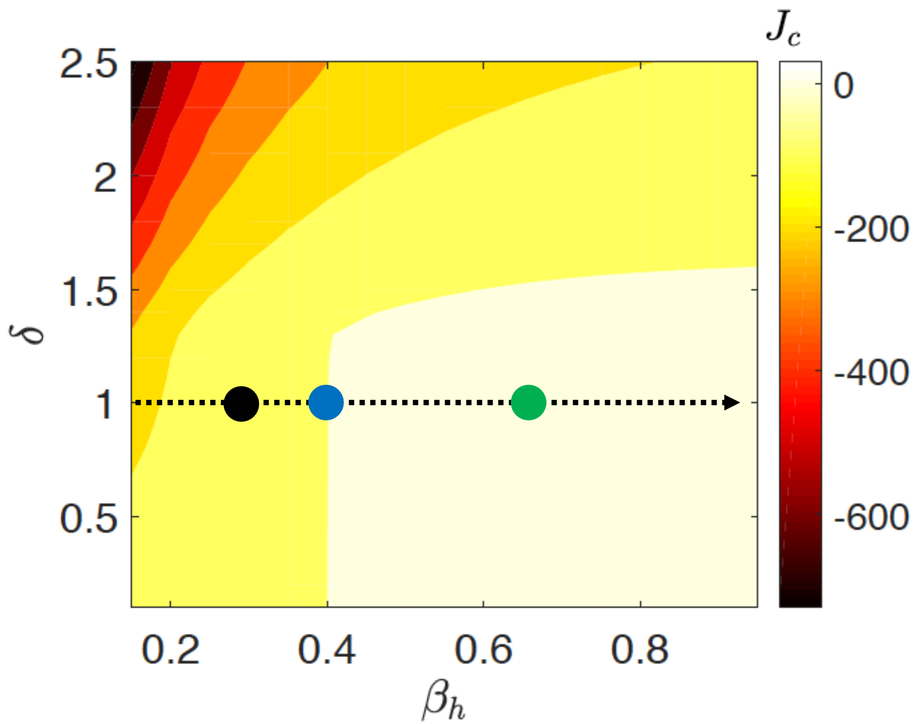
$\theta_c = 2, \theta_w = 4, \ \theta_h = 6$

$\beta_c = 1, \beta_w = 0.1$

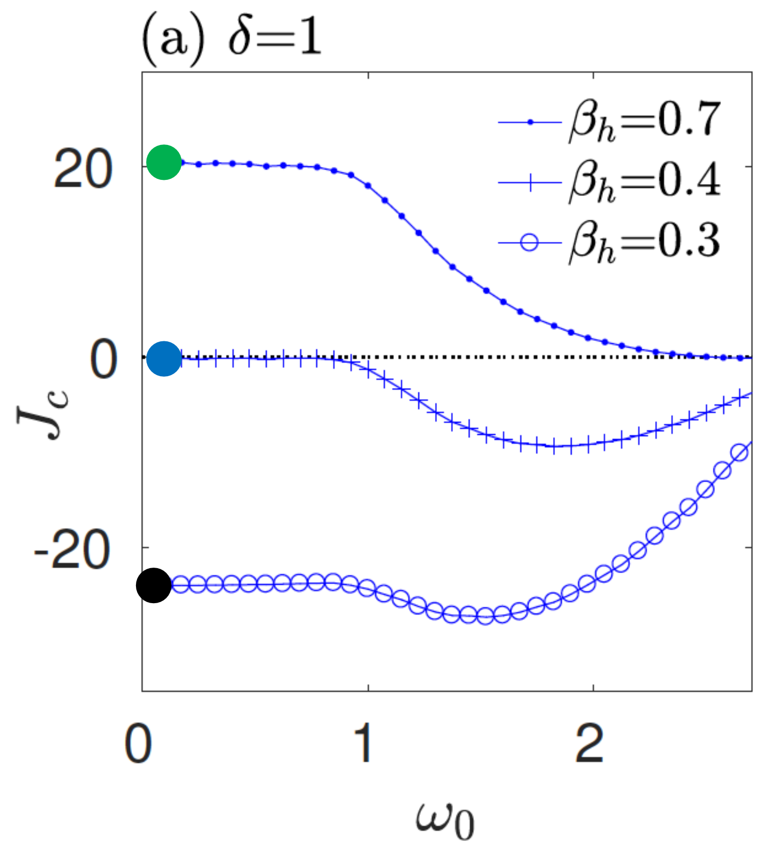
$\delta_c = 0.2$

$\delta_w = \delta, \ \delta_h = 2\delta$

Control

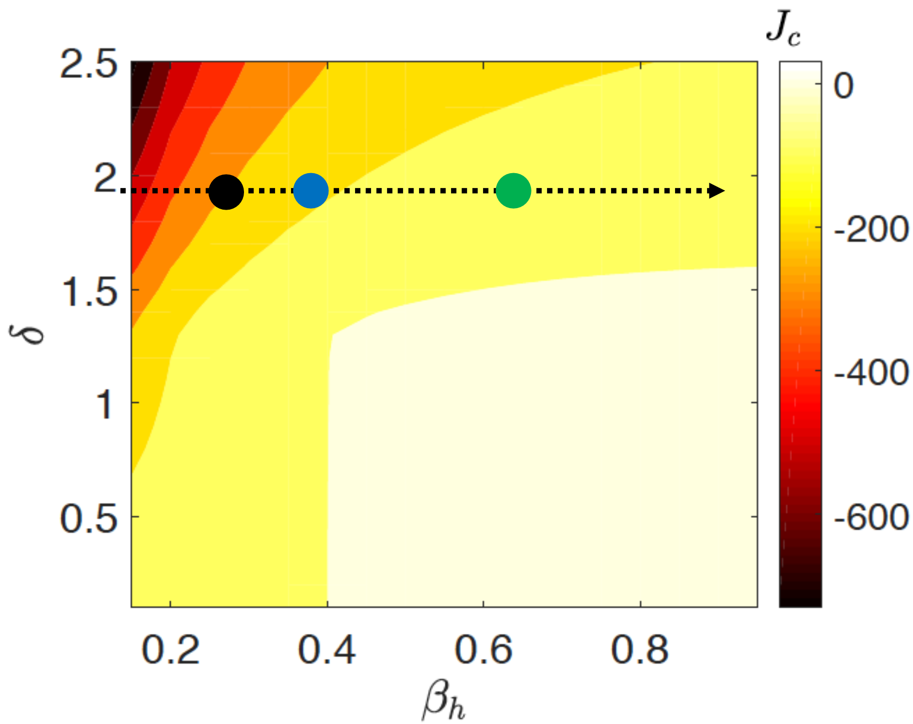


J_c
0
-200
-400
-600
2.5
2
1.5
1
0.5
δ
0.2
0.4
0.6
0.8
β_h

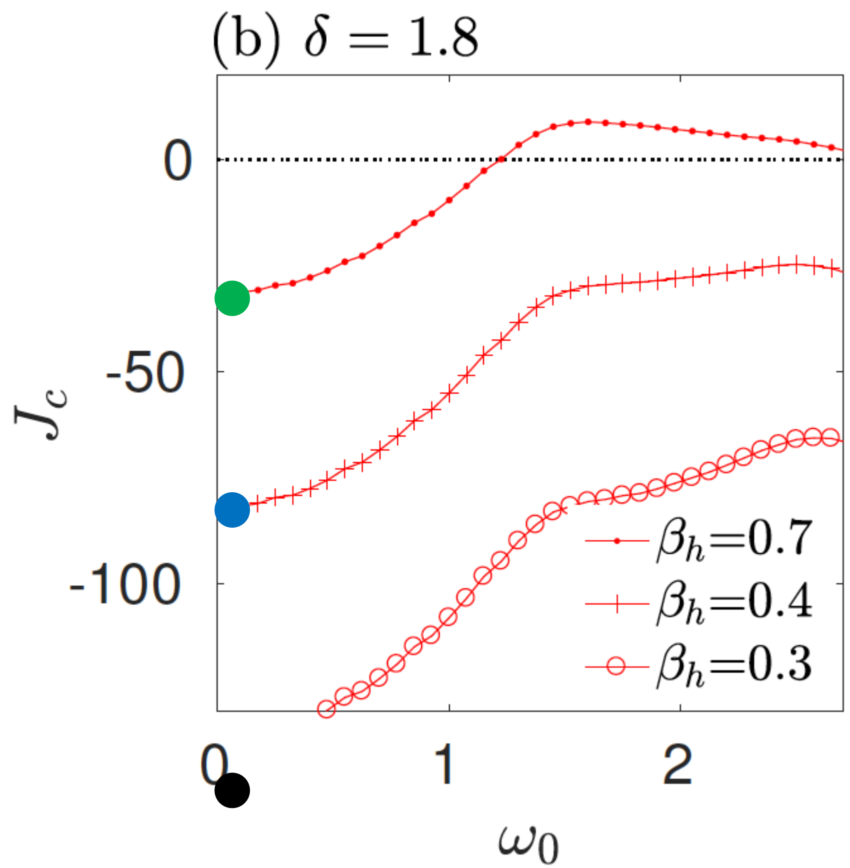


(a) δ=1
β_h=0.7
β_h=0.4
β_h=0.3
20
0
-20
J_c
0
1
2
ω_0

Control

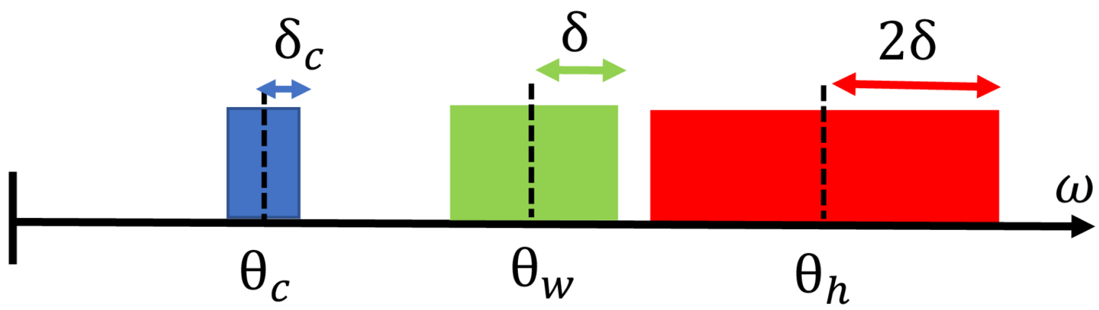


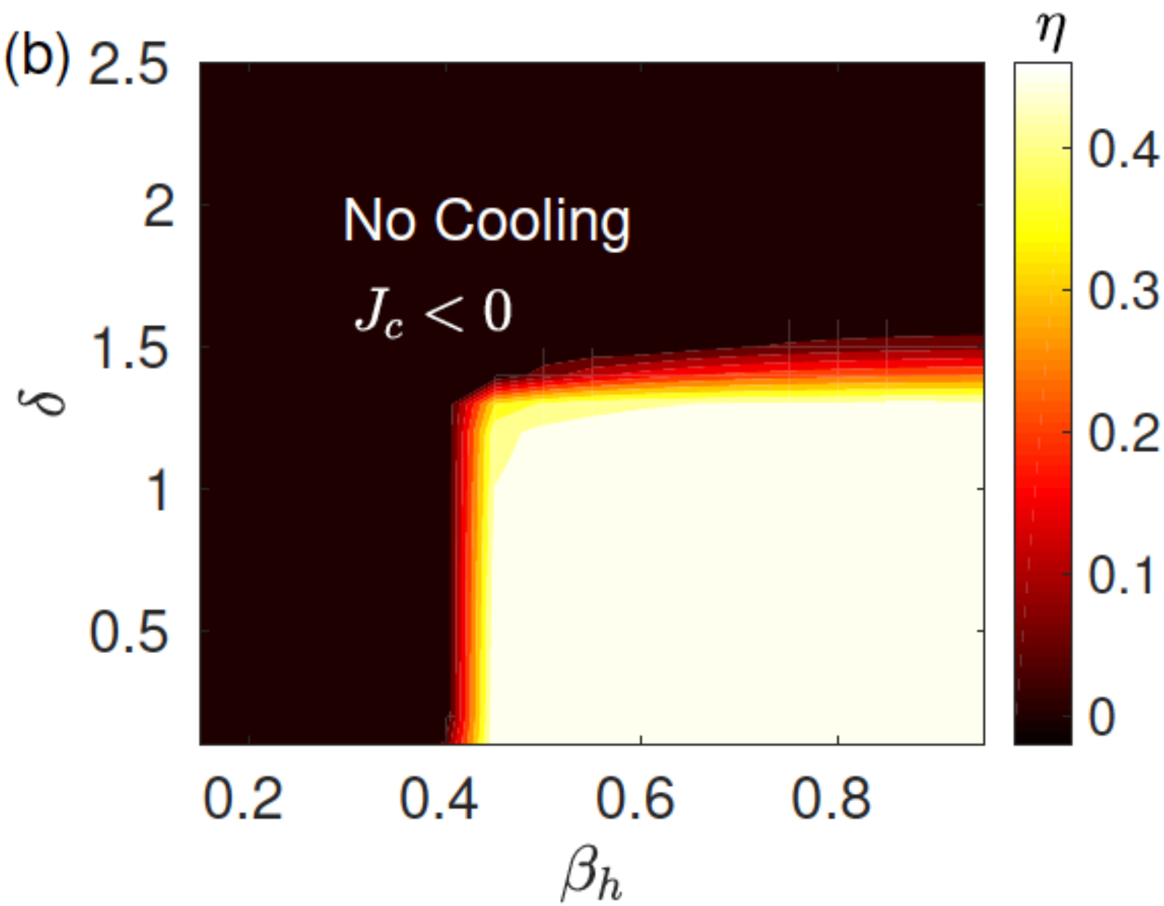
2.5
2
1.5
1
0.5
δ
0.2
0.4
0.6
0.8
β_h
J_c
0
-200
-400
-600



(b) $\delta = 1.8$
0
-50
-100
J_c
0
1
2
ω_0
β_h=0.7
β_h=0.4
β_h=0.3

COP $\eta \equiv \frac{J_c}{J_w}$





$\theta_c = 2, \theta_w = 4, \ \theta_h = 6$

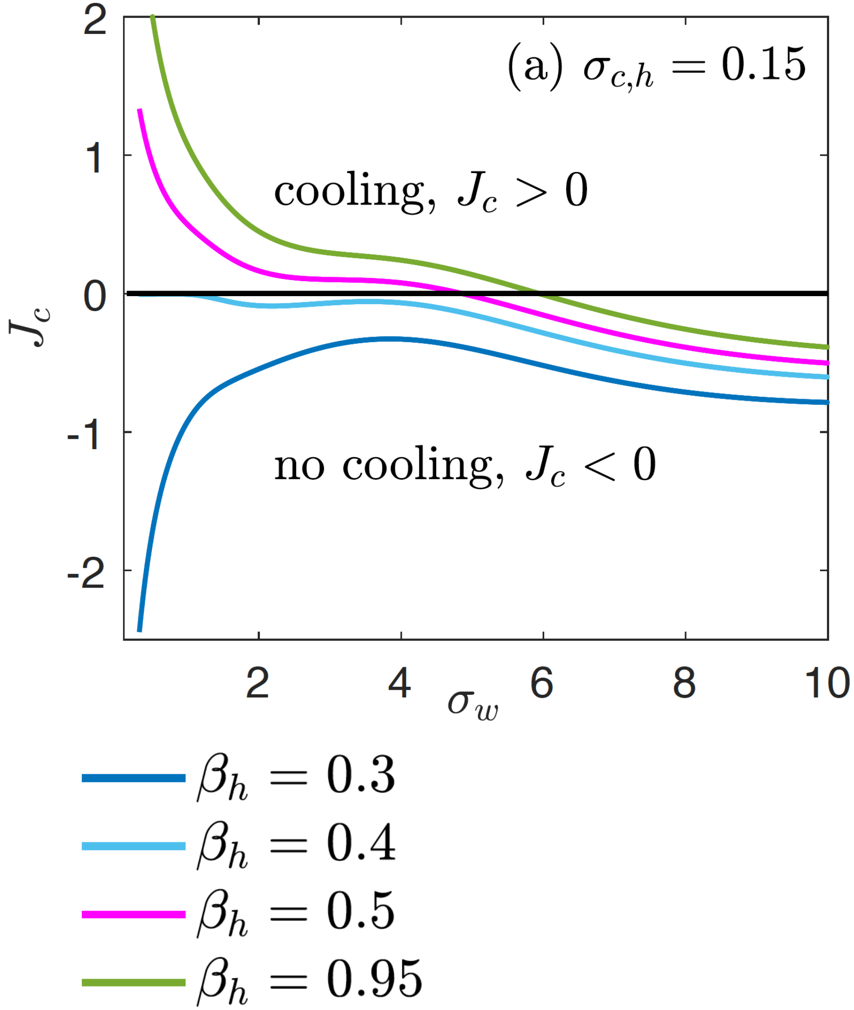
$\beta_c = 1, \beta_w = 0.1$

$\delta_c = 0.2$

$\delta_w = \delta, \ \delta_h = 2\delta$

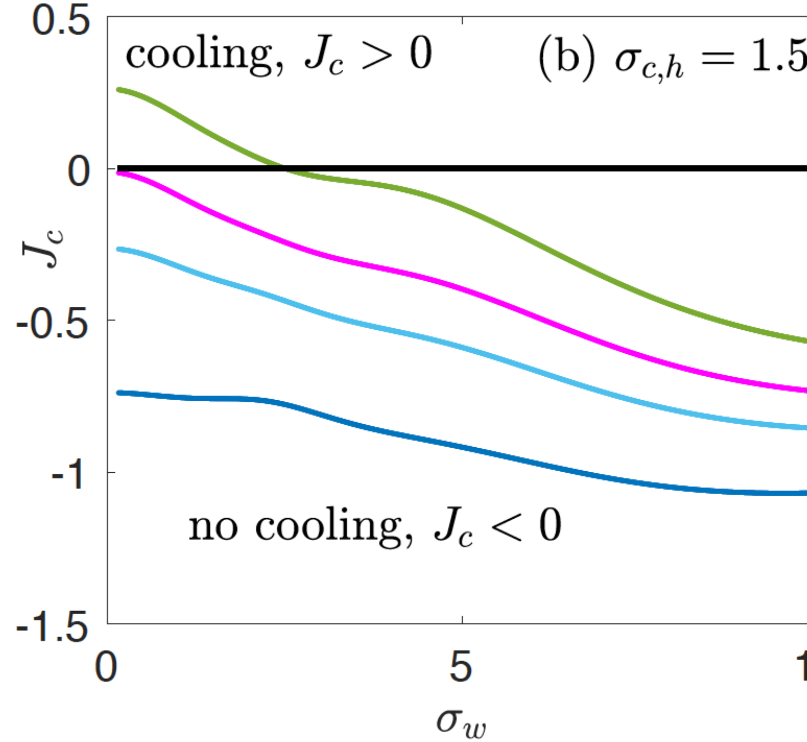
QAR with continuous Gaussian functions

$$M_\nu(\omega) = \frac{1}{\sigma_\nu}\left[e^{-(\omega-\theta_\nu)^2/2\sigma_\nu^2} + e^{-(\omega+\theta_\nu)^2/2\sigma_\nu^2}e^{\beta_\nu\omega}\right]$$



$\theta_c = 2, \theta_w = 4,\ \theta_h = 6$

$\beta_c = 1, \beta_w = 0.1$



Physical model

$$\hat{H} = \frac{\omega_0}{2}\hat{\sigma}_z + \sum_{\nu} \hat{H}_{B,\nu} + \gamma\frac{\hat{\sigma}_x}{2} \otimes \left(\hat{B}_c \otimes \hat{B}_h \otimes \hat{B}_w\right)$$

Model

$$\hat{B}_\nu = \sum_j \frac{\lambda_{j,\nu}}{\omega_{j,\nu}} \left(\hat{b}^\dagger_{j,\nu} + \hat{b}_{j,\nu}\right)$$

Two-time correction function

$$M_\nu(t) = \langle \hat{B}_\nu(t)\hat{B}_\nu(0)\rangle$$

$$\rightarrow M_\nu(\omega) = [n_\nu(\omega) + 1]\, g_\nu(\omega)$$

Bose-Einsetein occupation function

$$\boxed{n_\nu(\omega) = [e^{\beta_\nu \omega} - 1]^{-1}}$$

Spectral density

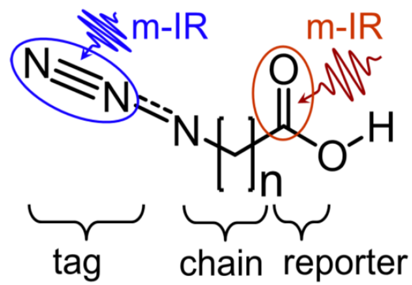
$$\boxed{g_\nu(\omega) = 2\pi \sum_j \frac{\lambda^2_{j,\nu}}{\omega^2_{j,\nu}} \delta(\omega - \omega_{j,\nu})}$$

Summary: FCS for quantum transport and thermodynamics

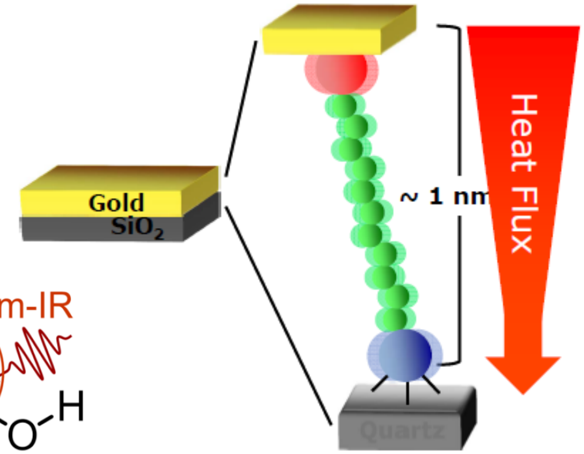
Full Counting Statistics: cumulants, fluctuation theorem, scaling laws

Method development:
Approximate → exact
(QME, NEGF, path integral)

Connection to experiments

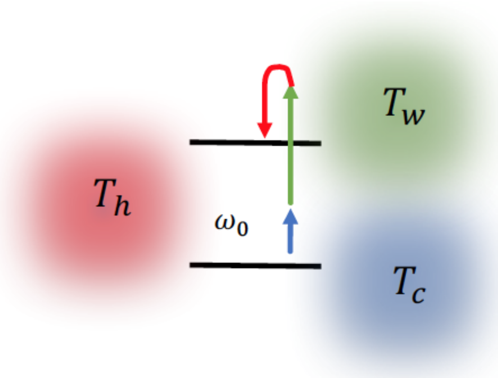


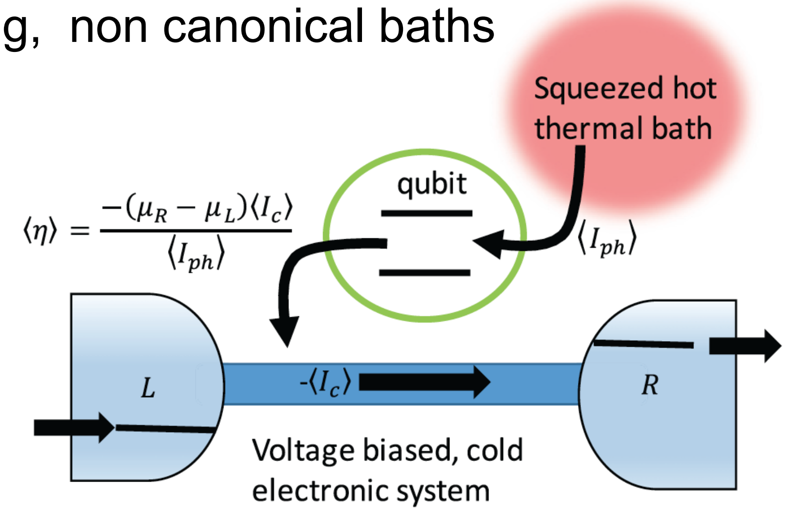
Rubtsov (Tulane)



Cahill (Illinois)

Quantum thermodynamics: strong coupling, non canonical baths





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