What is statistical mechanics?

- Explanation of macroscopic phenomena from microscopic information.
  - Example: Statistical thermodynamics
- Macroscopic measurements: crude level of description
  1. Measurements over regions of space containing many particles: required for statistics.
  2. Time scales typically long compared to molecular time scales: $\sim 10^{-15} - 10^{-18}$ sec.
  3. Generally focus on collective properties of system or average molecular properties (dynamics of a particle over long time scales.).
- Example: Local mass density of a gas in a region near point $\mathbf{r}$.
  - Can define $N_\Omega(\mathbf{r}, t)$ to be the instantaneous number of particles in volume $\Omega(\mathbf{r})$ at time t.
  - Number of particles in volume can fluctuate in time.
  - $N_\Omega(\mathbf{r}, t)$ is a *microscopic* variable: depends on the specific location of all particles.
  - Can divide up total volume into many small boxes whose dimension depends on the resolution of measurement.
  - Invent an "indicator" function for each volume $\Omega(\mathbf{r})$:

$$\Delta(\mathbf{r}_1 - \mathbf{r}) = \begin{cases} 1 & \text{if } \mathbf{r}_1 \in \Omega(\mathbf{r}) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

  - In limit of infinitely small boxes, $\Delta(\mathbf{r}_1 - \mathbf{r})$ is known as a "dirac delta function" (more later).
  - $\Delta(\mathbf{r}_1 - \mathbf{r})$ indicates if particle 1 is in box or not: depends on the location of particle 1.
  - Thus

$$N_\Omega(\mathbf{r}, t) = \sum_{i=1}^{N} \Delta(\mathbf{r}_i(t) - \mathbf{r}) \tag{2}$$

– Can measure the average of this instantaneous "box" density over a short time period

$$\begin{aligned} \rho_\Omega(\mathbf{r},t) &= \overline{N_\Omega(\mathbf{r},t)} \\ &= \int_{\Omega(\mathbf{r})} d\mathbf{r}' \sum_{i=1}^{N} \Delta(\mathbf{r}_i(t)-\mathbf{r}')P(\mathbf{r}',t) \\ P(\mathbf{r}',t) &= \text{probability of finding a particle at } \mathbf{r}' \text{ at time } t. \end{aligned}$$

– Using laws of dynamics, can obtain equations relating evolution of averaged quantities like local density, local fluid velocity, local temperatures and so on: Called the "hydrodynamic equations".

- Goal of course: Provide a solid foundation of concepts and methods of statistical mechanics.
- Need to review
  1. Basic math and statistics: probability theory.
  2. Basic physics: Newtonian and quantum dynamics.